Astron 400 Problem Set 7

Given: Oct 27. Due: Thursday, Nov 3 at the beginning of class

**Homework Policy:** You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other’s work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

**Reading:** Phillips Chapter 6

**Problem 1** Phillips 6.1

**Problem 2** Phillips 6.2

**Problem 3** Phillips 6.5

**Problem 4** Phillips 6.6

**Problem 5** GS: Supersonic Skydive

On Oct 14, 2012, Felix Baumgartner jumped from a high-altitude balloon and briefly went supersonic as he fell.

Assume that the atmosphere is well-described by an isothermal gas in hydrostatic equilibrium with $T = 273\, \text{K}$. At $z = 0$, it has $P = 101,000\, \text{Pa}$.

a. Show that both $P$ and $\rho$ decrease like $e^{-z/H}$ with a scale-height $H$ for a height $z$ above the ground. i.e.,

$$P = P_0 e^{-z/H}$$

and:

$$\rho = \rho_0 e^{-z/H}$$

b. Take $H = 7\, \text{km}$ to be the scale height. The sound-speed is defined here as:

$$c_s = \sqrt{\frac{k_B T}{m}}$$
so it is constant. The drag force is defined as:

\[ \frac{1}{2} AC_d \rho v^2 \]

with \( A \) the cross-sectional area of the falling body and \( C_d \) the drag coefficient (assume \( C_d = 1 \)). Equating this to gravity you get the terminal velocity:

\[ v_t = \sqrt{\frac{2mg}{\rho AC_d}} \]

We can define the terminal Mach number \( M_t \), which is the terminal velocity in units of the sound speed:

\[ M_t \equiv \frac{v_t}{c_s} \]

Use the ideal gas law and make other reasonable assumptions to determine the minimum height from which the jump could have occurred to have it be supersonic, i.e., \( M_t > 1 \).

c. Now, the jumper needs to start from a higher velocity in order to achieve the terminal velocity. Integrate the equations of motion (gravity and drag forces) to determine the velocity as a function of height starting from \( z = (20, 25, 30, 35, 40) \) km. For which of them is the jump actually supersonic (\( M = v(z) / c_s > 1 \))? For one of those, plot \( z \) on the y-axis against velocity on the x-axis. Show curves for the actual \( v(z) \) as well as the terminal velocity \( v_t(z) \) and the sound-speed \( c_s(z) \). Label the part of the curve where the jumper is supersonic.