Problem 1  Greenhouse Effect and Global Warming

Perhaps the main worry of our time is global warming. Here, we make a overly simple model of the greenhouse effect to get an idea of how numbers scale. In class, we showed that for an air-less Earth, one can derive an equilibrium temperature \( T_p = 255 \text{ K} \) (also, Eq. 19.5 of C&O), but this has to be modified when an atmosphere exists. Earth’s atmosphere is optically thin (nearly transparent) at visible wavelengths so the solar radiation hits the ground directly. However, the atmosphere is optically thick (opaque) at infrared wavelengths and absorbs the ground’s infrared black-body radiation. This heat is lost to space as the atmosphere radiates with a photospheric (top) temperature \( T = T_p \) (think why; hint: energy conservation).

a. Imagine the atmosphere as a single opaque layer with a uniform temperature \( T_p \). It is receiving heat from the ground (at temperature \( T_g \)) and radiates as much energy towards the ground as it radiates towards space. First ignoring the gradual warming of the atmosphere, use energy conservation to show that \( T_g = 2^{1/4} T_p \). Is the current ground temperature (288 K) colder or hotter than this?

b. A more sophisticated approach is to allow different layers in the atmosphere to have different temperatures, each emitting both upwards and downwards, with a constant net flux passing through. From this, one can derive (C&O, eq. 9.53) that the temperature will follow:

\[
T^4 = T_p^4 \left[ 1 + \frac{3}{4} \left( \tau - \frac{2}{3} \right) \right].
\]
where \( \tau \) is the infrared optical depth from the point being considered to the top of the atmosphere. (As discussed in C&O, the atmosphere emits at an effective optical depth \( \tau = 2/3 \).)

(b.1) Given \( T_g = 288 \, \text{K} \) and \( T_p = 255 \, \text{K} \), what is the atmospheric optical depth \( \tau_g \) to the ground?

(b.2) Also calculate \( \tau_g \) on Venus, given its no-atmosphere and actual ground temperatures.

(b.3) Supposing, simplistically, that the greenhouse effect scales linearly with \( \text{CO}_2 \), what is the expected rise in temperature on Earth as \( \text{CO}_2 \) is doubled from the current abundance?

c. A more accurate prediction requires simulations which consider all greenhouse gases (e.g., water vapour has more effect than \( \text{CO}_2 \)) and includes both positive and negative feedbacks as the earth’s temperature rises. Look up the “climate change 2007 synthesis report” from the intergovernmental panel on climate change, and find what they predict for a doubling of \( \text{CO}_2 \).