(Ground-based) Gravitational wave astronomy
Gravitational waves exist!

• Binary pulsars:

PSR B1913+16

Neutron Binary System
• separated by $10^6$ miles
• $m_1 = 1.44m_\odot$; $m_2 = 1.39m_\odot$; $\varepsilon = 0.617$

Prediction from general relativity
• spiral in by 3 mm/orbit
• rate of change orbital period

PSR 1913 + 16 -- Timing of pulsars

17 / sec

~ 8 hr
What are GW?

Really:
- Required by Einstein’s gravity
- “Ripples” on spacetime
- Often highly nonlinear

This talk:
- Linear, spin-2 transverse wave
- Cause “length changes”: \( h \sim \Delta L/L \)
- Like EM:

\[
\begin{align*}
 h_{xx} &= -h_{yy} \\
 h_{xy} &= -h_{yx}
\end{align*}
\]
Example: Two black holes (no spin)

**Amplitude:**

Two black holes

\[ f = 2 f_{\text{orb}} = 2 \left( \frac{\Omega}{\pi} \right) \]

\[ f = 10^3 \text{Hz} \left( \frac{M}{8M_0} \right)^{-1} \left( \frac{r}{6M} \right)^{3/2} \]

- **Characteristic relative length changes**
  \[ \sim \frac{\text{(kinetic energy)}}{\text{(distance)}} \]

\[ h \sim \frac{1}{d} \frac{d^2 I}{dt^2} \sim \frac{Mv^2}{d} \sim \frac{M}{d} \left( \frac{M}{8M_0} \right)^{5/3} \]

\[ h \sim 10^{-21} \left( \frac{M}{8M_0} \right)^{5/3} \left( \frac{d}{30Mpc} \right)^{5/3} \]

\[ \Delta L \sim h L \sim 10^{-21} \text{4km} \]
\[ \sim 4 \times 10^{-16} \text{cm} \]

LIGO

- laser light \( \sim 10^{-4} \text{cm} \)
- atom \( \sim 10^{-8} \text{cm} \)
What makes GW?

**Source:**
~any accelerating charge
screening limits size…

**Strong coupling:**
Imaging often practical:
(common sources)
>> wavelength
• Easy to make & detect
• Easy to obscure

**EM Waves**
Detection I: Scale important

LISA (planned)

Detectors

- Pulsar timing
- CMB fluctuations

Space-based interferometers (LISA)

Ground-based interferometers (LIGO/VIRGO/GEO/TAMA)

Sources

\[ f_{(Hz)} \]

\[ \lambda \text{(m)} \]

\[ 10^{-8} \]

\[ 10^{-4} \]

\[ 10^{10} \]

\[ 10^{4} \]

\[ 10^{6} \]

\[ 10^{8} \]

\[ 10^{10} \]

\[ 10^{12} \]

\[ 10^{10} \]

\[ 10^{2} \]

\[ \ldots \text{and more!} \]

Big bang

Merging Black Holes:
- Big (center of galaxy)
- Small (post-supernova)

Spinning neutron stars
Detection II: How sensitive?

Range:

Depends

(a) source (how much energy vs frequency): \( \frac{dE}{df} \)
(b) detector (preferred frequencies): \( S_h(f) \)

\[
D_{\text{burst}} \approx c^{-1} \frac{1}{\rho} \sqrt{\frac{2}{5\pi^2} \frac{\Delta E_{gw}(G/c^5)}{(f^2 S_h(f))}}
\]

\[D \approx \sqrt{\frac{L}{4\pi F_{\text{crit}}}}\]

Compare to flux threshold}

\[10^{52}\text{erg} (=\text{SN, all in GW})\]
What makes GW?

Example: Two black holes (no spin)

Waveform: 3 epochs

**Inspiral:**
- ~ Quasicircular orbits in potential $V(r \mid L(t))$
- Amplitude by changing binding energy

**Merger**
- Hard

**Ringdown:**
- One perturbed hole “ringing”

Movie: black-holes.org
Binaries: Chirp

- Frequency = 2*orbit...
- **Chirp**: Frequency, amplitude increase
  - Set by energy, energy loss rate
- **Identifying source**: Where on the track are we?
  Chirp rate, not frequency at any time
Measurables: Inspiral

- **Sky location:**
  \[ \delta \theta \simeq \lambda/d \]

  Wavelength set by **detector**: 100 Hz
  Detector d: 10 ms apart

  Easier for bright sources (as 1/amplitude)
  Easier with complex sources (spin & polarization): degeneracies

- **Mass**
  Via chirp rate
  \[ \text{df/dt} \rightarrow \text{mass} \]
  [mass ratio: fine structure]

- **Distance**
  \[ \text{SNR} \propto \frac{M^{5/6}}{d} \]

  **Absolute distance scale**
Bonus material: Binary “track” $h(f)$

“Amplitude” vs $f$: $h(f)$ \( \propto \sqrt{\frac{dE}{df} / f^2} \)

- Key concept: “Track” $h(f(t))$ for binaries
- Compare to detector sensitivities

D. Shoemaker, NSF review 2004
**Bonus material: Supernova to BH**

**Complex signal, several epochs** [bounce; unstable NS; collapse; ringdown]

Mostly high frequency (collapse time $\sim R/c$~ms), messy

Ott Dec 2010