A closed-form model for gravitational waves from precessing BH-NS binaries

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Based on: Lundgren and O’Shaughnessy [arxiv:1304.3332]
How do binaries evolve when radiating?

- Shrinking binary “chirps” through quasicircular orbits

\[ \frac{d\Phi}{dt} = \frac{v^3}{M} \quad \leftrightarrow \quad \nu = (M\omega_{orb})^{1/3} \]

\[ \frac{dt}{dv} = \frac{E'(\nu)}{-\mathcal{F} + \partial_v M(\nu)} \]

...all power series in \(v\)
A helpful way to represent the radiation

- Want $\tilde{h}(f|\lambda)$ and $\frac{d\tilde{h}(f|\lambda)}{d\lambda}$

- Stationary-phase ("F2"): example calculation

$$h(t) = (-2)Y_{2,2}(\hat{n})h_{2,2}(t) + (-2)Y_{2,-2}(\hat{n})h_{2,-2}(t)$$

$$h_{2,2}(t) \propto \frac{\eta v^2}{d_L} e^{-2i\Phi(t)}$$

$$\tilde{h}_{2,2}(\omega) \propto \frac{\eta v^2}{d_L} \frac{1}{\sqrt{id^2\Phi/dt^2/\pi}} e^{i\Psi(\omega)}$$

$\Psi(\omega)$: power series in $v$

set by stationary phase condition (=Legendre transform of $\Phi$)

$$\Psi = \omega t - 2\Phi$$

$$\frac{d\Psi}{d\omega} = t \quad \frac{dt}{d\omega} = \frac{dt}{dv} \frac{dv}{d\omega}$$
How do precessing binaries evolve?

- Orbit shrinks
  - Spin angular momentum more significant
  - L<\(S\) easier if unequal mass or high spin

\[ J = L + S_1 + S_2 \]

Example: BH-NS (40 Hz)
Representing precessing kinematics

- Minimize power series to preserve physics

\[ \frac{d\hat{L}}{dt} \approx \frac{J}{r^3} \left( 2 + \frac{3m_2}{2m_1} \right) \times \hat{L} \]

\[ |J| = |L + S| \] not globally fit by a power series when \( L \sim S \)

keep unexpanded

- Extend known single-spin precession solutions
  - \( \beta(v) \): set by \( |L| \) and (conserved) \( L.S \)
  - \( \alpha \): precession phase
    \[ \Omega_p \frac{dt}{dv} d\nu \]

integrated can be done, including \( \beta(v) \)

to adequate PN order

- Related integrals also can also be evaluated
Representing GW from precessing binaries

• **Time-domain signal**

\[
h_+(t) - i h_\times(t) = e^{-2i\psi} \sum_{lm} h_{lm}(t) - 2Y_{l,m}(\theta, \phi)
\]

\[
= e^{-2i\psi} \sum_{lm} \sum_{m'} D_{m',m}^l(\alpha(t), \beta(t), \zeta(t)) h^{\text{ROT}}_{l,m}(t) - 2Y_{l,m'}(\theta, \phi)
\]

\[\overleftrightarrow{R(t)}\]

• **Fourier-transform term-by-term**

\[
X(t) \equiv D_{m',2}^l(R(t)) \times \frac{\eta v^2}{dL} e^{-i2\Phi(t)} \times (-2) Y_{l,m'}(\theta, \phi)
\]

\[
\tilde{X}(\omega) \simeq D_{m',2}^l(R(t(\omega))) \times \frac{\eta v^2}{dL} \frac{e^{i\Psi(\omega)}}{\sqrt{-id^2\Phi/dt^2}/\pi} \times (-2) Y_{l,m'}(\theta, \phi)
\]

• **Regroup terms: “carrier+sideband”** [restricted to \((l,m)=(2,2) + (2,-2)\)]
What’s in the paper

- **Time domain form**

\[
h_+ = \frac{2M \eta}{D} v^2 \text{Re} \left[ \sum_m z_m e^{im\alpha} e^{2i(\Phi-\zeta)} \right]
\]

\[
z_m = -2Y_{2,m}(\beta,0) \frac{4\pi}{5} \left[ e^{-2i\psi} -2Y_{2m}(\theta,0) + e^{2i\psi} -2Y_{2-m}(\theta,0) \right].
\]

- **Kinematics**

\[
\gamma \equiv \frac{|S_1|}{|L|} = \left( \frac{m_1 \chi}{m_2} \right) v;
\]

\[
\Gamma_J \equiv \frac{|J|}{|L|} = \sqrt{1 + 2\kappa \gamma + \gamma^2}.
\]

- **Precession angles**

\[
\alpha(v) = \eta \left( 2 + \frac{3m_2}{2m_1} \right) \int v^5 \Gamma_J \left( \frac{dt}{dv} \right) dv
\]

\[
\zeta(v) = \eta \left( 2 + \frac{3m_2}{2m_1} \right) \int v^5 (1 + \kappa \gamma) \left( \frac{dt}{dv} \right) dv.
\]

- **Frequency domain form**

\[
\bar{h}_+(f) \simeq \frac{2\pi M_c^2}{D} \sqrt{\frac{5}{96\pi}} (\pi M f)^{-7/6} \sum_m z_m e^{i(\Psi-2\zeta)+im\alpha}
\]
Conclusions

• Model to represent GW from BH-NS binaries
  • Good approximation to known time-domain waveform
  • Easier to use, computationally and analytically
  • “Sideband” interpretation of previous results [Brown et al 2012]

• Applications pending
  • Parameter estimation [MCMC, Fisher, ...]
  • Searches: model performance, identify targeted improvements
  • Other:
    • guide precessing inspiral-merger-ringdown models
    • framework for double-spin analysis (e.g., perturbative)
Related developments

• Implemented, applied
  • Implemented in LIGO software; some tests with parameter estimation

• Recent work
  • Systematic separation of timescales [Klein et al, arxiv:1305.1932]
    • Similar physics, exhaustive detail. [Unnecessarily complicated waveform]
  • Inspiral-merger-ringdown model [Hannam et al arxiv:1308.3271]
    • Very similar inspiral physics, plus ad-hoc merger ringdown
    • Our opinion: missing some degrees of freedom...but easily fixed
Sample applications

- Amplitude
  - set by geometry of "precession cone"
  - model: average z over precession cycle

\[
\int_0^{T_{\text{prec}}} \frac{dt}{T_{\text{prec}}} |z|^2
\]
How well does it do?

- Full physics not implemented in v0 code (e.g., amplitude)
- Still... good fit
\[ 
\psi_{(F_2)}(f) = 2\pi ft_c - \phi_c + \frac{3}{128\eta} v^{-5} \left\{ 1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right) v^2 + (4\beta - 16\pi)v^3 \\
+ \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right) v^4 + \left(\frac{38645}{756}\pi - \frac{65}{9}\pi\eta - \gamma\right) (1 + 3\log(v)) v^5 \\
+ \left[ \frac{11583231236531}{4694215680} - \frac{640}{3}\pi^2 - \frac{6848\gamma_E}{21} - \frac{6848}{21}\log(4v) + \left(\frac{15737765635}{3048192} + \frac{2255\pi^2}{12}\right) \eta \\
+ \frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 + 160\pi\beta - 20\xi \right] v^6 + \pi \left(\frac{77096675}{254016} + \frac{378515}{1512}\eta - \frac{74045}{756}\eta^2 \right) \\
+ \beta \left(\frac{43939885}{254016} + \frac{259205}{504}\eta + \frac{10165}{36}\eta^2\right) - \gamma \left(\frac{2229}{112} - \frac{99}{4}\eta - 20\xi\right) v^7 \right\} 
\]

[e.g., Nitz et al 2013]