What can we learn about the neutron-star equation of state from inspiralling binary neutron stars?

Ben Lackey, Les Wade
What can we measure from the waveform?

- Phase shift during the inspiral from tidal interactions
  - Tidal field $\mathcal{E}_{ij}$ of one NS induces quadrupole moment $Q_{ij}$ in other NS
    \[ Q_{ij} = -\lambda(\text{EOS}, M_{\text{NS}})\mathcal{E}_{ij} \]
    \[ -\Lambda(\text{EOS}, M_{\text{NS}})M_{\text{NS}}^5\mathcal{E}_{ij} \]
- Increased quadrupole moment leads to more tightly bound system and additional quadrupole radiation
What can we measure from the waveform?

- Tidal effects first appear at same order as 5PN point-particle terms
- Leading term is a linear combination of the tidal deformabilities for each NS
- Results in a phase shift of $\sim 1$ cycle up to ISCO depending on the EOS

$$\tilde{h}(f) \propto \frac{\mathcal{M}^{5/6} f^{-7/6}}{D_L} e^{i \psi(f)}$$

$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128 \eta v^5} \left[ 1 + (\text{PP-PN}) - \frac{39}{2} \tilde{\Lambda}(\mathcal{M}, \eta, \text{EOS}) v^{10} \right]$$

$$\tilde{\Lambda} = \frac{8}{13} \left[ (1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta(1 + 9\eta - 11\eta^2)}(\Lambda_1 - \Lambda_2) \right]$$
Available physical models for EOS can be accurately fit by a parametrized piecewise polytrope

Parametrized piecewise polytrope reproduces neutron star properties to a few percent
Estimating EOS parameters from LIGO data

• Analogue of 2-step procedure described by Steiner, Lattimer, Brown (Astrophys. J. 722, 33) for mass-radius measurements

  • They combined several mass-radius measurements from accreting neutron stars to estimate EOS parameters

  • We will use estimates of $M-\eta-\tilde{\Lambda}(M, \eta, \text{EOS})$ from several BNS inspiral events to estimate EOS parameters
Step 1: Estimate $\mathcal{M} - \eta - \tilde{\Lambda}(\mathcal{M}, \eta, \text{EOS})$

- Can estimate BNS parameters from Bayes theorem:

$$p(\hat{\theta}|d_n, \mathcal{H}, \mathcal{I}) = \frac{p(\hat{\theta}|\mathcal{H}, \mathcal{I}) p(d_n|\hat{\theta}, \mathcal{H}, \mathcal{I})}{p(d_n|\mathcal{H}, \mathcal{I})}$$

- $\hat{\theta} = \{\alpha, \delta, \iota, \psi, D_L, t_c, \phi_c, \mathcal{M}, \eta, \tilde{\Lambda}\}$
- $d_n$: gravitational wave data from nth BNS system
- $\mathcal{H}$: waveform model
- $\mathcal{I}$: prior information about the parameters

- Posterior calculated with MCMC (see Les Wade’s talk) or estimated with Fisher matrix which assumes Gaussian likelihood and prior

- Marginalized distribution trivial to compute with MCMC or Fisher

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n, \mathcal{H}, \mathcal{I}) = \int p(\hat{\theta}|d_n, \mathcal{H}, \mathcal{I}) d\theta_{\text{marg}}$$
Step 2: Estimate EOS parameters

- Can estimate EOS parameters from Bayes theorem:

\[
p(\bar{x}|d_1 \ldots d_N, \mathcal{H}, \mathcal{I}) = \frac{p(\bar{x}|\mathcal{H}, \mathcal{I})p(d_1 \ldots d_N|\bar{x}, \mathcal{H}, \mathcal{I})}{p(d_1 \ldots d_N|\mathcal{H}, \mathcal{I})}
\]

- \(\bar{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, M_1, \eta_1, \ldots, M_N, \eta_N\}\)

- \(d_1 \ldots d_N\): gravitational wave data from all \(N\) BNS events

- prior: Flat in EOS parameters except \(v_s = \sqrt{dp/d\epsilon} \leq c\) and \(M_{\text{max}} \geq 1.93 M_\odot\)

- Likelihood:

\[
p(d_1, \ldots, d_N|\bar{x}, \mathcal{H}, \mathcal{I}) = \prod_{n=1}^{N} p(M_n, \eta_n, \tilde{\Lambda}_n|d_n, \mathcal{H}, \mathcal{I})|\tilde{\Lambda}_n = \tilde{\Lambda}(M_n, \eta_n, \text{EOS})
\]

- Perform MCMC simulation over the 4+2N parameters then marginalize over the 2N mass parameters
Simulating a population of BNS events

- We chose the “true” EOS to be MPA1
  - Moderate EOS in middle of parameter space
  - $R(1.4M_\odot) \sim 12.5\text{km}$, and $M_{\text{max}} \sim 2.5M_\odot$

- Sampled 50 BNS systems with SNR > 8
  - Individual masses distributed uniformly in $(1.2M_\odot, 1.6M_\odot)$
  - Sky position and distance distributed uniformly in volume
  - Orientation distributed uniformly on unit sphere
  - $\tilde{\Lambda}$ then calculated from masses and “true” EOS
EOS Parameters

- 1 and 2-parameter marginalized distributions
- 2-parameter contours represent 95% confidence

True values

1-d marginalized PDFs

2-d marginalized PDFs

log\((p_1)\) (dyne/cm\(^2\))

1 BNS
5 BNS
20 BNS
50 BNS
True EOS
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- Chain of EOS parameters from MCMC simulation gives histogram of pressures for each density
- 95% confidence interval shown for each density
EOS function $p(\rho)$

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NS Radius and Tidal Deformability

- Del Pozzo et al. found $\lambda(1.4M_\odot)$ can be measured to +/-10% with 50 sources but the slope of $\lambda(M)$ cannot be measured.

- Fitting the EOS function $p(\rho)$ instead of $\lambda(M)$, and using known EOS properties and mass constraints, dramatically improves the measurement of $\lambda(M)$ as well as any other quantity derived from the EOS.

- Radius, love number, moment of inertia, upper limit on spin, ...

Simulated masses uniformly distributed in this range.
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![Graph showing simulated masses uniformly distributed in this range and plots of $R$, $\lambda$, and $\lambda M^3$ vs. $M$ for different cases.](image-url)
MCMC instead of Fisher matrix

- Beginning work using MCMC for both individual BNS parameter estimation and EOS parameter estimation (with Les Wade)
- Single inspiral at 100Mpc with network SNR of 30

68% confidence
95% confidence
99.7% confidence
True EOS
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Conclusions

• Detailed EOS information can be found from the inspiral of BNS systems with aLIGO
  
  • Errors in pressure between $\sim 10\%$ and factor of 2 depending on density using 50 BNS systems
  
  • Corresponds to errors in radius of $\sim 0.5\text{km}$ and error in tidal parameter of $\sim 10\%$
  
• Systematic errors from inexact waveform templates will be the primary difficulty in measuring the EOS with BNS inspirals
Extra Slides
Current EOS constraints

- **Causality**: Speed of sound must be less than the speed of light in a stable neutron star \( v_s = \sqrt{dp/d\epsilon} < c \)

- **Maximum mass**: EOS must be able to support the observed star with mass greater than \( 1.93M_\odot \)

- Mass-radius measurements also provide strong constraints (Steiner et al.)
Parametrized EOS

- Each exact measurement of $\mathcal{M}-\eta-\tilde{\Lambda}(\mathcal{M}, \eta, \text{EOS})$ restricts parameter space to $N-1$ dimensions.

- Multiple observations with different NS masses can measure all 3 parameters if accurate enough.

  - Most BNS systems will have maximum central density below $10^{15} \text{ g/cm}^3$, so $\Gamma_3$ is not constrained by inspiral observations.