Probing Strong Field Physics with Gravitational Waves

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What is the “Strong Field”? 

It depends!!
Strong Relative to What?    Relative to Solar System and Binary Pulsar Gravity

Pragmatic Definition: Region of Spacetime where **Non-Linear and Dynamical Gravity** is Important

Define a “characteristic scale”

\[ c \equiv \frac{M}{D} \]
Typical Mass of the System Typical Dynamical Distance

Example: Late Inspiral and Coalescence of neutron stars and/or black holes. \( D \) here is the orbital separation, so \( C \) is of order unity. For the double binary pulsar, \( C \) is about \( 1 \times 10^{-5} \) -> not strong-field.

Other Examples: Mountains on pulsars and supernovae. But these are less “clean” due to messy nuclear physics.
What are the most important discoveries?

(i) FIRST DETECTION !!

(ii) Mapping properties of compact binary populations.

(ii) Studying fundamental properties of compact objects.

(ii) Testing General Relativity in the strong field.

But testing GR is a waste of time with LIGO-like instruments, because we don’t have the sensitivities, the signals are not long enough, etc.

FALSE !! (this talk)

Testing GR is (at least) as important as other discoveries after making the first GW detection.
But Why Bother Probing the Strong Field?

(i) **Waste Argument**: We hope that in this decade accurate measurements of strong-field gravity will be available. It would be a waste not to use this data to test GR.

(ii) **Arrogance Argument**: There could be surprises; perhaps it’s too arrogant of us to believe that we know everything.

**Availability and Potential Discovery**

In the 1800s, Newtonian gravity was the standard model, without any experimental evidence to suggest its incompleteness. Yet we know today that this theory is wrong, thanks to advances in gravitational experiments. One such “new windows” are opened, we should allow ourselves to be surprised.
Simplifications for this Talk

(i) Focus on Gravitational Waves (see Stairs for EM)
   They travel unimpeded/unobscured from source to us
   They carry detailed info about the gravitational field/interaction.

(ii) Focus on Binary Coalescences
   Most studied to date in the alternative theory context.
   Sample the strongest and most dynamical gravitational fields
   (see Hayama’s talk for tests with burst sources)

(iii) Focus on Late, Quasi-Circular Inspirals (inside 100 M)
   Easiest to understand physically and mathematically.
   Generalizable to more complicated scenarios.
Road Map

I. How do we Probe the Strong-Field?

II. What does a Deviation look like?

III. How do we Implement Generic Tests?
Part I: How to Probe...
A Proposed Recipe

(1) For low SNR sources, GWs are buried in noise. Construct **Templates** and extract via **matched filtering**, assuming GR is right. After all, Solar System/Binary Pulsar Tests have confirmed GR in the weak-field limit, so the early inspiral must be right.
Solve the Einstein Equations and compute the gravitational wave (metric) perturbation

\[ h_{x,+}(t) = A_{x,+}(t) \cos[\Phi(t)] \]

Solution is always the product of:

1. A time (or freq) dependent amplitude that eg. depends on the chirp mass, the luminosity distance, the inclination angle, etc.
2. The cosine of a time (or freq) dependent phase that eg. depends on the chirp mass, mass ratio and spins.
A Proposed Recipe

(1) For low SNR sources, GWs are buried in noise. **Construct Templates** and extract via **matched filtering**, assuming GR is right. After all, Solar System/Binary Pulsar Tests have confirmed GR in the weak-field limit, so the early inspiral must be right.

(2) Go back to your data and study whether you have missed something or whether the data is consistent with GR:

- **If it is consistent**
  - **Test GR**
  - Place a constraint on how large Phase and Amplitude deviations could be given uncertainties.

- **If it is not consistent**
  - **Characterize any**
  - Cross-Correlate with other detectors to eliminate inst. and astroph. artifacts
  - Phase or Amplitude deviation. Trace back to a specific modification to GR.
Part II: What does a Deviation look like?
Example of a Simple Deviation

Give a mass to the graviton and gravitational waves will not travel at the speed of light, but instead (assuming Special Rel.)

\[ \frac{v^2_g}{c^2} = 1 - \frac{m^2_g c^4}{E^2} \]

If coincident EM/GW detection, then difference in time of arrival translates immediately into a bound on the graviton mass.

A pure GW detection can still bound \( m_g \). The graviton’s \( E (=h f) \) increases during inspiral (low to high freq. chirp). Thus, its speed also changes inducing a shorter than expected time of passage of a given number of cycles (phase correction).

\[ \tilde{h}(f) = \tilde{A}_{GR}(f) e^{i\Psi_{GR}(f)} e^{-ic_0 \frac{D M_{e}}{X^2_9} (M_e f)^{-1}} \]

(Will '98)
**Other Examples**

(i) Scalar-Tensor theories:
(Will '94, Scharre & Will '02, Will & Yunes '04, Sorri, Suenanno & Will '05, Yagi & Tanaka '09)

\[ \tilde{h} = \tilde{h}_{\text{GR}} e^{i \beta_{\text{BD}} \eta^{2/5} u^{-7/3}} \]

- because of dipolar energy emission
- inversely related to the BD coupling parameter
- reduced GW frequency
  \( u = \pi M_c f \)

(ii) Massive Graviton Theories:
(Will '96, Will & Yunes '04, Stavridis & Will '09, Arun & Will '09, Yagi & Tanaka '09)

\[ \tilde{h} = \tilde{h}_{\text{GR}} e^{i \beta_{\text{MG}} \eta^0 u^{-1}} \]

- related to graviton Compton wavelength

(iii) Gravitational Parity Violation:
(Alexander, Finn & Yunes '08, Yunes, O'Shaughnessy, Owen, Alexander '10)

\[ \tilde{h} = \tilde{h}_{\text{GR}} (1 + \alpha_{\text{PV}} \eta^0 u^1) \]

- related to CS coupling

(iv) G(t) theories:
(Yunes, Pretorius, Spergel '10)

\[ \tilde{h} = \tilde{h}_{\text{GR}} \left( 1 + \alpha \dot{G} \eta^{3/5} u^{-8/3} \right) e^{i \beta_{\text{G}} \eta^{3/5} u^{-13/3}} \]

- related to \( \dot{G} \)
(v) Quadratic Gravity

\[ \tilde{h} = \tilde{h}_{\text{GR}} e^{i \beta_{\text{QG}} \eta^{-4/5} u^{-1/3}} \]

because it’s a higher curvature correction related to theory couplings

(Yunes & Stein, ’11)

(vi) Extra-Dimensions:

\[ \tilde{h} = \tilde{h}_{\text{GR}} e^{i \beta_{\text{EG}} \eta^{3/5} u^{-13/3}} \]

related to size of extra dimension

(Inoue & Tanaka ’03, Yagi, Tanahashi & Tanaka ’11)

We have still not found any theories whose predicted gravitational wave correction cannot be mapped to such a phase and Amp corrections
Part III: How do we Implement Such Generic Tests?
Analogy to ppN

In 1970’s, the parameterized post-Newtonian scheme was developed to cure an outbreak of alternative theories w/Solar System Exp.

ABC of ppN:

A) Expand the field equations about Minkowski
B) Assuming a perfect fluid source and a PN expansion, solve the field equations in terms of Green function potentials
C) Construct a generalization of the metric (a “super-metric”) in terms of ppN potentials and ppN parameters.

\[
g_{ij} = \delta_{ij}(1 + 2\gamma U)
\]

Why not try the same with waveforms?
You cannot test GR by assuming GR templates a priori

Promote the response function to a non-GR response, with parameters that control “well-motivated” deformations

Extremely Simple Eg:
Inspiral ppE template

GR: \((a, a, \beta, b) = (0, 0, 0, b)\)

BD: \((a, a, \beta, b) = (0, \alpha_{BD}, -7/3)\)

PV: \((a, a, \beta, b) = (\alpha_{CS}, 1, 0, b)\)

[see Rodriguez’s talk for another ppE implementation in terms of quadrupolar (no-hair theorem) deviations]

\[ h = \tilde{h}_{GR} (1 + \alpha \eta^c u^a) e^{i \beta \eta^a u^b} \]

Match filter with this new response function and let the data decide what these ppE parameters are.
Properties of ppE

1) Reproduces all known alternative theory predictions for the GW phase of inspiraling binaries (e.g., Brans-Dicke, massive graviton).

2) Reproduces generic (model-independent) deviations in the Hamiltonian and in the radiation-reaction force.

3) Includes tests of the no-hair theorem as a special case.

4) In given limit, ppE reduces to PN tests.

\[ \Psi_{ppE} = \Psi_{GR}(1 + \beta f^b) \rightarrow \frac{3}{128} (M f)^{-5/3} (1 + \beta f^{2/3}). \]

5) Extended to merger/ringdown deformations (not shown).

6) Extendable to the time-domain (in progress). (Yunes and Pretorius '09)
Questions for ppE

Given a GW detection, how sure are we it was a GR event?
Statistically significant anomalies in the signal?

Can we test for deviations from/consistency with GR, without explicitly building templates banks for all conceivable theories?

How would we mischaracterize the universe if GR was close but not quite the correct theory of nature? (“fundamental bias”)

<table>
<thead>
<tr>
<th>Templates/Theories</th>
<th>GR</th>
<th>ppE</th>
</tr>
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<tbody>
<tr>
<td>GR</td>
<td>Business as usual</td>
<td>Quantify the likelihood of GR being the underlying theory describing the detected event, within the class of alt. theories captured by ppE</td>
</tr>
<tr>
<td>Not GR</td>
<td>Understand the bias that could be introduced filtering non-GR events with GR templates</td>
<td>Measure deviations from GR characterized by non-GR ppE parameters.</td>
</tr>
</tbody>
</table>
Constraining GR Deviations

GR Signal/ppE Templates, Projected constraints, SNR = 20

\[ \tilde{h} = \tilde{h}_{GR} \left( 1 + \alpha f^a \right) e^{i \beta f^b} \]

Strong Field

Weak Field

LISA

LIGO (for 3:1 mass ration binary)

(Yunes and Hughes, '10)

(Cornish, Sampson, Yunes & Pretorius in prep.)
Parameter Bias

Non-GR Signal/GR Templates, SNR = 20

Non-GR injection, extracted with GR templates (red) and ppE templates (blue). GR template extraction is “wrong” by much more than the systematic (statistical) error. “Fundamental Bias”

(Cornish, Sampson, Yunes & Pretorius in prep.)
Identifying GR Deviations
Non-GR Signal/ppE Templates, SNR = 20

Filter an injected ppE signal \((a,\alpha,b,\beta) = (0,0,.5,.5)\) with a ppE template family. The marginalized posterior for \(\beta\) clearly shows a preference away from GR (away from \(\beta = 0\)). LIGO

You can also compute the Bayes factor as a function of \((b,\beta)\). You would find a strong preference (BF > 10) for \(b = 0.5\) and \(\beta > 0.3\).

(Cornish, Sampson, Yunes & Pretorius in prep.)
Conclusions

A generic and model-independent (ppE) framework has been proposed to carry out such tests.

Preliminary studies suggest Earth and Space GW detections could be used to search for generic deviations away from GR.

If no deviations are found, one could place the strongest constraints on GR yet in the non-linear, dynamical (strong-field regime).

If deviations are found, one could then try to trace back what is sourcing such a deviation.

The full exploit of GW astrophysics will require the strong collaboration between relativists, astrophysicists, data analysts & high-energy theorists.
Correlations

peak at $a=0$ is degeneracy between luminosity distance and effective $\alpha$ (LISA example)

bump at $b=5/3$ (PN value) is a partial degeneracy between chirp mass and $\beta$ (LIGO example)

peak at $b=0$ is degeneracy between phase of coalescence and $\beta$ (LISA example)
Resonances

At certain “resonant” exponents, you cannot distinguish between GR and an alternative theory modification (spikes).
(degeneracies not sampled in the previous plot)
Bayes Factor

Odds ratio for ppE signal injection at different values of beta and (a, alpha, b)=(0,0,.5). Extraction with ppE template. Suggests beta > 0.3 can easily be observed.
Detecting GR deviations

Bayes factor for injection with $b = 0.5$, varying beta.
Can we Listen for GR deviations?

Since gravitational waves are oscillatory functions, with chirping frequency, we can convert them into sound files!

Consider a neutron star in a quasi-circular orbit around a $1 \times 10^5$ Msun supermassive BH in GR.

Non-GR gravitational waves chirp differently!!

Consider the same theory but in an alternative theory of gravity (eg. one where Newton’s constant is time-varying).

The difference between the GR and non-GR waves sounds like this:
But What Theory Do We Pick?

A Minimal (?) Set of Criteria:

1. Weak-Field Consistency (existence and stability of physical solutions, satisfaction of precision tests).
2. Strong-Field Inconsistency (deviations only where experiments cannot currently rule out modifications)

It's not easy to fool Mother Nature! (Wald)

Other Nice Criteria:

3. Well motivated from fundamental physics.
4. Well-posed theory ?? This is hard to do...
Classification of Approaches

(1) **Top-Down Approach.** Construct a “natural” theory from divine/ethereal inspiration. Study observational consequences of this theory.

**Advantage.** Complete analytic control over the theory.

**Disadvantage.** Consider single theory. Relies on divine inspiration

(2) **Bottom-Up Approach.** Construct an experiment and search for deviations from canonical beliefs. (Not because you know a deviation must be present, but because you want to test your working model)

**Advantage.** Search over wide class of deviations.

**Disadvantage.** Generic deviations hard to map to single theory.
Bottom-Up Approach

Inspiration: “It doesn’t matter how beautiful or “natural” your theory is, or how smart you are, or what your name is. If it disagrees with experiment, it’s wrong. That’s all there is to it.

Null Tests versus Deviation Search

Null Test. Assume GR is right and observe its predictions. Arun, et. al. PRD 74 (2006), also see talk by Favata.

Deviation Search. Assume “small” deviations from GR are possible and constrain their magnitude. Contains Null Tests. Yunes & Pretorius, PRD 80 (2009), also see talk by Vigeland.
# Top-Down Accuracy Studies

<table>
<thead>
<tr>
<th></th>
<th>Will</th>
<th>Scharre, Will</th>
<th>Will, Yunes</th>
<th>Berti, Buonanno, Will</th>
<th>Arun, Will</th>
<th>Stravridis, Will</th>
<th>Yagi, Tanaka</th>
<th>Ajith, Keppel</th>
<th>Solar System</th>
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</thead>
<tbody>
<tr>
<td><strong>Binary Mass</strong></td>
<td>x</td>
<td>1.4:E3</td>
<td>1.4:E3</td>
<td>x</td>
<td>x</td>
<td>1.4:E3</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><strong>BD Coupling Par. (e4)</strong></td>
<td>x</td>
<td>24</td>
<td>20</td>
<td>10</td>
<td>x</td>
<td>x</td>
<td>0.7</td>
<td>x</td>
<td>4</td>
</tr>
<tr>
<td><strong>Binary Mass</strong></td>
<td>1E7:1E6</td>
<td>x</td>
<td>1E6:1E6</td>
<td>1E6:1E6</td>
<td>2E6:1E7</td>
<td>1E6:1E6</td>
<td>1E7:1E6</td>
<td>5E7:5E7</td>
<td>x</td>
</tr>
<tr>
<td><strong>Graviton Compton Wavelgth (e21 cm)</strong></td>
<td>6.3</td>
<td>x</td>
<td>3.1</td>
<td>1.33</td>
<td>5</td>
<td>4</td>
<td>3.1</td>
<td>52</td>
<td>0.00028</td>
</tr>
<tr>
<td><strong>Details</strong></td>
<td>First MG study, no spin</td>
<td>First ST study, no spin</td>
<td>As a func. of Det.</td>
<td>non-prec., spinning</td>
<td>amp. corr.</td>
<td>spin + prec</td>
<td>spin + prec + ecc</td>
<td>IMR</td>
<td>Cassini, 3rd Law Solar Sys</td>
</tr>
</tbody>
</table>

*Notes:*
- x indicates presence.
- Values represent specific parameters or mass values.