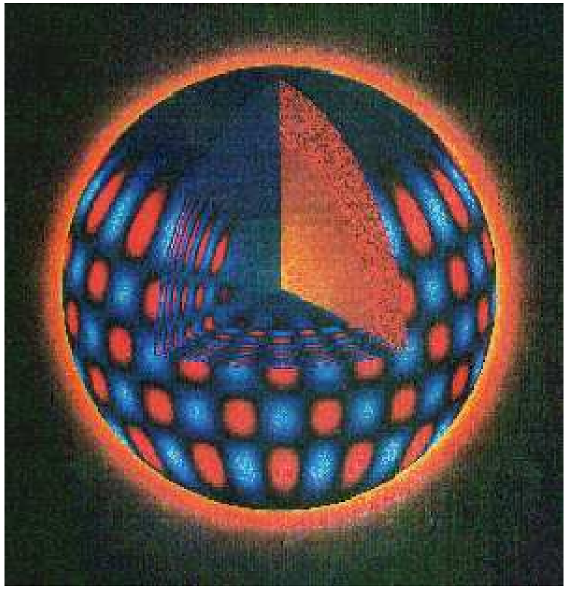
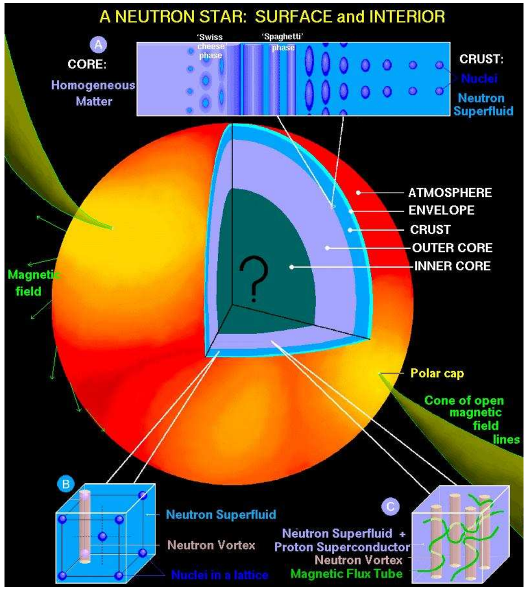
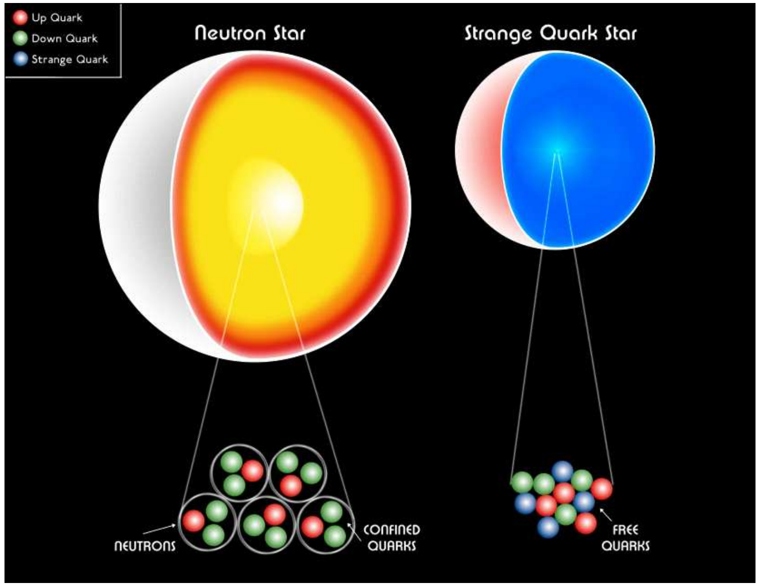
Gravitational waves from oscillations of compact stars: what can we learn from current observations?







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Introduction

- Oscillating compact stars are promising sources of gravitational radiation.
- Interesting upper limits on gravitational-wave (GW) burst emission were set recently by the LIGO and Virgo Collaborations:
  - ▷ associated with a Vela timing glitch,
  - ▷ in coincidence with electromagnetic triggers from magnetars.
- Those searches targeted f-modes of neutron stars with normal equations of state.

- Exotic equations of state predict solid quark matter cores and are much more interesting from the GW point of view.
  - ▷ Recent continuous-wave searches constrained the true ellipticities of some pulsars to be many orders of magnitude less than the theoretically maximum sustainable ones.
- Corequakes could/would also produce GWs, most likely transients.
- In this talk, we discuss the relevance of the corequake scenario on the detection of gravitational waves.

Short Reminder

Upper limits of recent GW burst searches:
- $E < 5.0 \times 10^{44}$erg (Vela glitch [1])
- $E < 1.4 \times 10^{47}$erg (SGR 0501+4516 burst [2])

Fundamental modes (f-modes) of conventional neutron stars (NSs):
- ringdown signals, $h \sim \sin 2\pi f t \cdot \exp(-t/\tau)$
- frequencies $f \sim 1 - 3$kHz, damping times $\tau \sim 50 - 500$ms

Ellipticity constraints from GW continuous-wave (CW) searches:
- $\epsilon < 10^{-4}$ (for Crab), $\epsilon < 7 \times 10^{-8}$ (for J2124-3358) [3]

Theoretical predictions for exotica (with solid interiors):
- $\epsilon < 10^{-4}$ [4, 5, 6]

Cores of solid:
- neutron matter, were much favoured in the '70s [7]
- quark matter, were proposed recently [8, 9]

shear modulus: $\mu_{\rm core} \simeq 10^{32-33}{\rm erg/cm}^3 \simeq 10^3 \mu_{\rm crust}$

Detection Prospects

The (conservative?) "f-modes from crustquakes" scenario:

- Detection Distances: $D < 10 \left(\frac{E}{10^{43}\mathrm{erg}}\right)^{1/2} \left(\frac{2\mathrm{kHz}}{f}\right) \left(\frac{10^{-21}\mathrm{Hz}^{-1/2}}{h_{\mathrm{rss}}}\right)$ pc
  ▷ 10^{45}erg to reach the closest pulsar at 100pc, 10^{47}erg for a magnetar at 1kpc
- Target Source Sample: $T < 10^8$yr (glitching pulsars)
- Event Rates (per source): $R < 1/3$yr (Vela rate)
- Number of Detectable Sources: $N < \frac{3}{100}\mathrm{yr}^{-1} \cdot T \cdot \frac{D^3}{100\mathrm{kpc}^3}$ (uniform distribution)
- Expected Detections (per year): $N \cdot R \sim 10^{-2} \ll 1$

The (optimistic?) "s-modes from corequakes" scenario:

- better sensitivity at lower frequencies ($f \cdot h_{\mathrm{rss}} : 2 \cdot 10^{-21} \longrightarrow 5 \cdot 10^{-22}\mathrm{kHz} \cdot \mathrm{Hz}^{-1/2}$?)
- more energetic events ($E : 10^{43} \longrightarrow 10^{45}$erg?)
- larger target sample ($T : 10^8 \longrightarrow 10^{10}$yr?) and/or higher event rates ($R : 1 \longrightarrow 10\mathrm{yr}^{-1}$?)
- overall ($N \cdot R > 1$?)

What are the s-modes?

▷ Oscillation modes supported by regions of NSs with non-vanishing shears ($\mu \neq 0$, shear modes).

▷ For example, quasi-periodic oscillations (QPOs) in the x-ray tails after magnetar flares are usually explained as (torsional) shear oscillations of the crust.

Parameters of Expected Signals

The periods of core torsional oscillations would be (Schumaker & Thorne 1983, [10]):

$$\text{Period} = 2\pi/\omega \simeq 2\pi R_\mathrm{c}/v_\mathrm{s} \sim 0.3\mathrm{ms} \tag{1}$$

and their damping times (current quadrupole emission) would be:

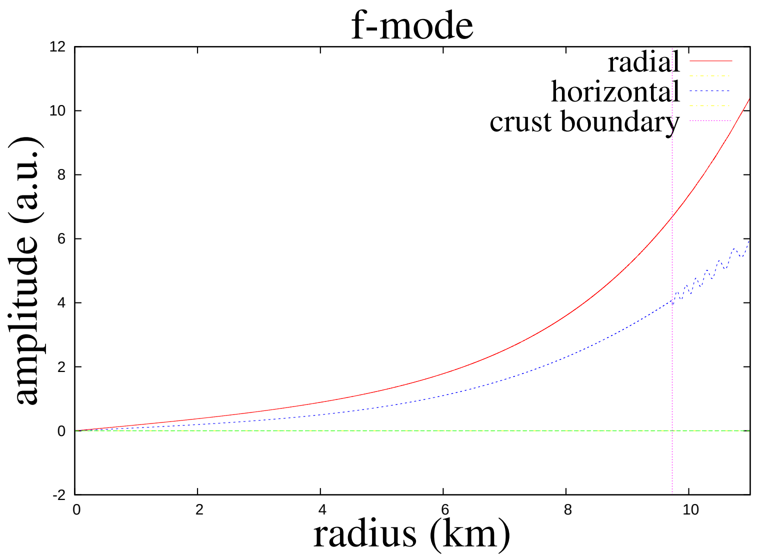
$$\tau \sim 30 \left(\frac{GM_\mathrm{c}}{R_\mathrm{c}c^2}\right)^{-1} \left(\frac{v_\mathrm{s}}{c}\right)^{-5} \omega^{-1} \sim 1\mathrm{s} \tag{2}$$

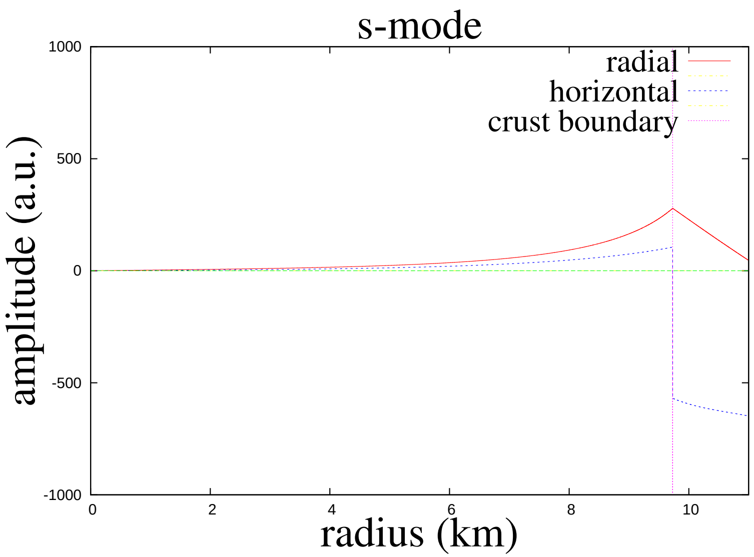
where
R_c is the core radius (< 7km),
M_c is the core mass ($< 1M_\odot$),
v_s is the speed of shear waves ($< 10^{10}$cm/s).

- The frequencies implied in (1) will be gravitationally redshifted by several per cent.
- Similar expressions for spheroidal oscillations (mass multipole emission).
- More detailed calculations are needed!
  - ▷ Straightforward solutions of Einstein equations, e.g. equations of Schumaker & Thorne (1983), [10].
  - ▷ A posteriori estimations, e.g. formula of Balbinski & Schutz (1982), [11].

Numerical Results:

matter motions and expected GW signals from crustquakes





Crust			f	τ
thickness	1.53km	f-mode	2.6kHz	50ms
mass	$0.1M_\odot$	s-mode	39Hz	25d
at densities	$< 2 \cdot 10^{14}\mathrm{g/cm}^3$			
shear modulus	$10^{30}\mathrm{erg/cm}^3$			

▷ Inefficient GW emitters (low densities and nonrelativistic speeds)

See also McDermott, Van Horn & Hansen 1988, [12]

Numerical Results:

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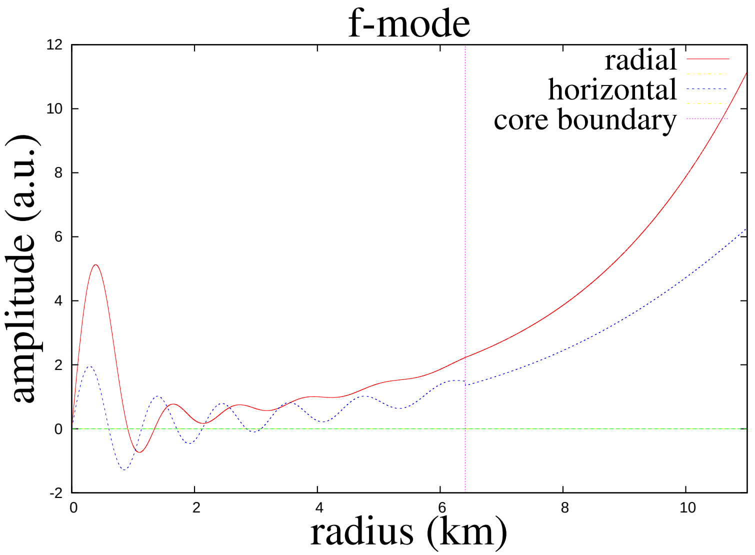
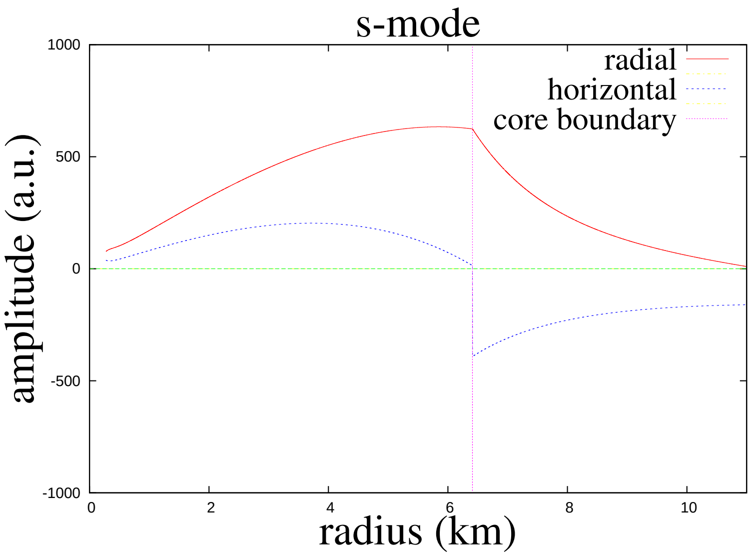




Core			f	τ
thickness	6.41km	f-mode	2.6kHz	50ms
mass	$0.85 M_{\odot}$	s-mode	176Hz	2h
at densities	$> 1 \cdot 10^{15} \mathrm{g/cm}^3$			
shear modulus	$10^{33} \mathrm{erg/cm}^3$			

- ▷ For smaller cores ($R_{\rm c} \downarrow$), $f \uparrow$ $\left(\propto R_{\rm c}^{-1}\right)$ and $\tau \downarrow$ $\left(\propto f^{-4}\right)$, (cf. with numbers (1)-(2))
- ▷ Calculations were done with many approximations (quantitatively very uncertain)

Energetics: the good news

Release of strain energy in a corequake (e.g. Shapiro & Teukolsky 1983, [13]):

$$\Delta E = 2\left(A + B\right)\left(\epsilon_0 - \epsilon\right)\Delta\epsilon \sim 10^{45}\mathrm{erg} \tag{3}$$

where
$A\left(\sim GM^2/R\right), B\left(\sim \mu R_\mathrm{c}^3\right)$ depend on the stellar model ($\sim 10^{53}$erg),
$\epsilon_0 - \epsilon$ is the reference minus actual oblateness at the time of the quake ($\sim 2 \cdot 10^{-3}$),
$\Delta\epsilon$ is the actual oblateness decrease ($\sim 10^{-6}$).

Rates

Time between corequakes:

$$t_\mathrm{q} \simeq T\frac{\omega_\mathrm{q}^2}{\Omega^2}|\Delta\epsilon| \quad \text{where} \quad T \equiv \frac{\Omega}{\dot{\Omega}} \quad \text{and} \quad \omega_\mathrm{q}^2 \equiv \frac{2A^2}{BI_0} \tag{4}$$

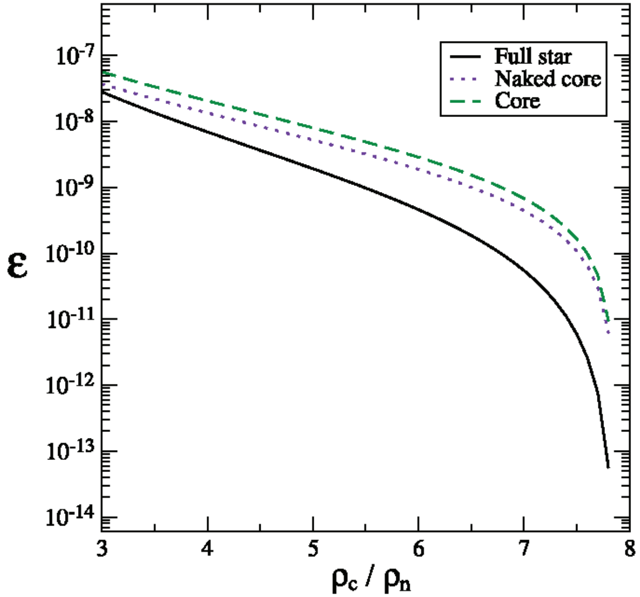
- would be shorter than time between crustquakes ($B \uparrow$)
  ▷ for Vela ($T \sim 10^4\mathrm{y}, \Omega \sim 100\mathrm{Hz}, I_0 \sim 10^{45}\mathrm{gcm}^2$) : $t_\mathrm{q} \sim 200\mathrm{y}$
- but would be long(er) for old(er) NSs ($T \uparrow, \Omega \downarrow$)

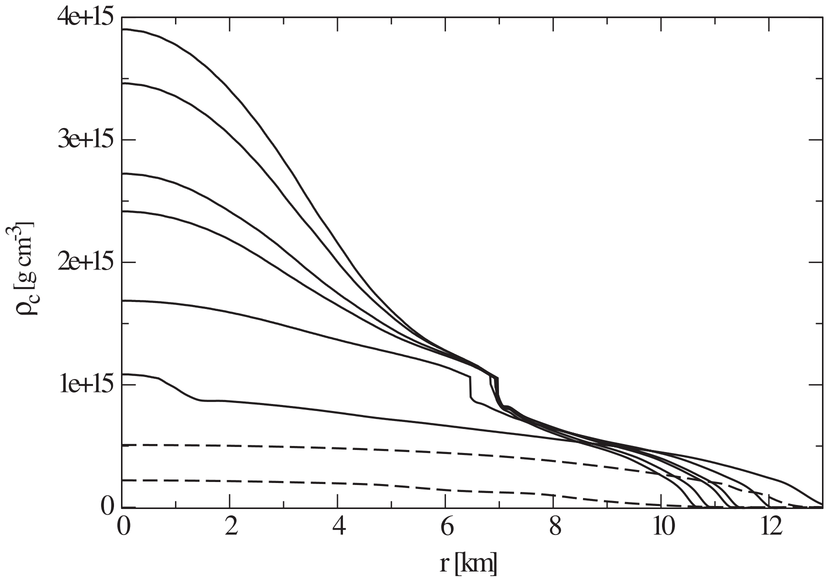
Energetics: the bad news

The presence of a fluid (envelope) can reduce the total energy released (Baym, Lamb & Lamb 1976, [14]):

$$\Delta E \lesssim 5 \cdot 10^{44}\,(A_{\rm c})_{52} \left(\frac{\delta\rho}{\rho_{\rm c}}\right) \left(\frac{\epsilon_0}{10^{-2}}\right) \left(\frac{\Delta\epsilon_0}{10^{-6}}\right) \text{erg} \tag{5}$$

where ρ_c is the core density and $\rho_c - \delta\rho$ is the fluid density.





(Left figure taken from [5]; density jumps would be favourable also for CW searches)
(Right figure taken from [6]; it could be $\delta\rho/\rho_{\rm c} \ll 1$)

Future Directions

We need to refine our models of corequakes:
- no significant theoretical progress for 30 years!

We need to address more rigorously the following questions:
- how much (GW) energy would be released?
- which modes would be excited? how the GW signals would look like?
- how core bulk matter motions (isotropic GW emission) would couple to external magnetic fields (beamed EM emission)?

We need to give our best shot on data analysis:
- More sensitive all-sky NS ringdown (burst) searches?
  - ▷ matched filtering could help!
- Extended signal parameter space?
  - ▷ f down to $\sim$ 100Hz? longer τ (hours to days)?
  - ▷ computationally feasible?
- NS targeted all-times searches?
  - ▷ we could miss EM triggers (no timing observations, beaming effects, quiescent phases)
  - ▷ EM and GW emission could be not correlated (weak magnetic fields, weak core-crust couplings)

References

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[6] Knippel & Sedrakian, http://arxiv.org/abs/0901.4637

[7] Pandharipande, Pines & Smith, http://adsabs.harvard.edu/abs/1976ApJ...208..550P

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[9] Mannarelli, Rajagopal & Sharma, http://arxiv.org/abs/hep-ph/0702021

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[14] Baym, Lamb & Lamb, http://adsabs.harvard.edu/abs/1976ApJ...208..829B