Using astrophysical knowledge in gravitational-wave data analysis of binary inspirals

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Outline

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   - Gravitational waves
   - LIGO/Virgo

2. GW parameter estimation
   - Signal and noise
   - Markov-chain Monte Carlo algorithm
   - Example SPINPSPIRAL analysis
   - MCMC examples
   - Analysis of a BH-NS signal
   - Analysis of a BH-BH signal
   - The importance of having spins

3. Using astrophysical information
   - Example: GRB without spin
   - Example: GRB with spin

4. Conclusions
Gravitational waves

GWs:

- "Ripples in spacetime"
- Predicted by Einstein's theory of General Relativity
- Indirectly observed for the Hulse-Taylor binary pulsar:

(Ertem et al., Science, 2008)
Gravitational waves

- propagate transversely at the speed of light
- are quadrupole radiation at the lowest order
- stretch and squeeze spacetime in two polarisations
- allow us to measure their amplitude

Strain: \[ h(t) = h_+(t)F_+(t) + h_\times(t)F_\times(t) = \frac{\delta L(t)}{L} \sim 10^{-22} \]
Inspiral waveforms with increasing spin

LIGO and Virgo detect the last $\sim 10$ s of a binary inspiral:

$$a_{\text{spin}} \equiv S/M^2 = 0.0, 0.1 \text{ and } 0.5$$
Signal injection into detector noise

Example:
- Using two 4-km detectors H1, L1
- Inject signal coherently
- $\Sigma \text{SNR} = 17$
- Retrieve physical parameters using MCMC
Use Markov-Chain Monte Carlo for parameter estimation
Follow-up after detection
Gaussian, stationary noise or LIGO/Virgo detector data
Analyse software injections, hardware injections, detection candidates/interesting events
Include spin in injections and analysis
Use any network composed of LIGO/Virgo detectors:

\[
\text{PDF}(\vec{\lambda}) \propto \text{prior}(\vec{\lambda}) \times \prod_i L_i(d|\vec{\lambda})
\]

Result: posterior probability-density function (PDF) of the parameter set that describes the model (9–12–15 D)
SPIN_SPIRAL example

\[ M (M_\odot) \]

Signal: 2.994
Median: 2.967
\[ \Delta_{95\%} \]: 2.72%

Iteration: 4.59E+06
Data points: 3.08E+05
Correlations increase with spin

Parameters:
- BH-NS
- H1 & L1
- $M_1 = 10\ M_\odot$
- $M_2 = 1.4\ M_\odot$
- $a_{\text{spin}} = 0.1, 0.8$
- $\theta_{\text{SL}} = 55^\circ$
- Network SNR $\approx 25$
MCMC results for the analysis of a BH-NS signal

Parameters:
- $H1, L1, V$
- $M = 10, 1.4 \, M_\odot$
- $d_L = 22.4 \, \text{Mpc}$
- $a_{\text{spin}} = 0.8$, $\theta_{SL} = 55^\circ$
- $\Sigma \, \text{SNR} \approx 17.0$
- simulated noise

van der Sluys et al., 2008
Sky position for signals with different spins

Spinning BH, non-spinning NS:
$10 + 1.4 \, M_\odot$, 16–22 Mpc, $\Sigma$ SNR=17

2 detectors, $a_{\text{spin}} = 0.0$
2-σ accuracy: $821^\circ\!2$

2 detectors, $a_{\text{spin}} = 0.5$
2-σ accuracy: $163^\circ\!2$

3 detectors, $a_{\text{spin}} = 0.5$
2-σ accuracy: $40^\circ\!2$

van der Sluys et al., 2008; Raymond et al., 2009; Poster by Ben Farr
Analysis of a BH-BH signal with spins

van der Sluys et al., in preparation

HS-2:
- 3.5-pN waveform
- 3 detectors (H1,L1,V)
- $M = 7.6 \, M_\odot$
- $\eta = 0.238$
- $M_1 = 11.0 \, M_\odot$
- $M_2 = 7.0 \, M_\odot$
- $a_{s1,2} = 0.9, 0.7$
- $\theta_{s1,2} = 10, 20^\circ$
- $d_L = 74.5 \, \text{Mpc}$
- $\Sigma \text{SNR}=15$
- simulated noise
The importance of having spins in your analysis

Signal with spins

Analysis with spinning template

Analysis with non-spinning template

van der Sluys et al., in preparation
Using astrophysical data to constrain parameters

1 detector (H1):

- **M (M_☉)** distribution:
  - 1.162, 1.164, 1.166, 1.168
- **η** distribution:
  - 0.2, 0.22, 0.24
- **d_L (Mpc)** distribution:
  - 0, 5, 10, 15, 20

- **M_1 (M_☉)** distribution:
  - 1.5, 2
- **M_2 (M_☉)** distribution:
  - 0.8, 1, 1.2
- **ζ (°)** distribution:
  - 0, 50, 100, 150

3 detectors (H1,L1,V):

- **t_c (s)** distribution:
  - 0, 0.01, 0.02, 0.03
- **d_L (Mpc)** distribution:
  - 0, 10, 20, 30
- **ζ (°)** distribution:
  - 0, 100, 150
- **ψ (°)** distribution:
  - 0, 50, 100, 150

**NS-NS, non-spinning:**
1.2 + 1.5 M_☉

\[ d_L \approx 10.2 - 17.8 \text{ Mpc} \]

(\(\Sigma\) SNR=15.0)

- No astrophysical information
- Sky position known
- Sky position and distance known

van der Sluys et al., in preparation

See also: Nissanke et al., 2010
Using astrophysical data to constrain parameters

2 detectors (H1,L1):

BH-NS, spinning BH:
$10.4 \pm 1.4 M_\odot$
$d_L \approx 20.2 \text{ Mpc}$
($\Sigma \text{SNR}=15.0$)

No astrophysical information

Sky position known

Sky position and distance known

van der Sluys et al., in preparation
Conclusions

**SPINspiral**

- SPINspiral can recover the 12–15 parameters of a binary inspiral, including one or two spins, using an MCMC technique.

- Sky-position reconstruction (few $\times 10^2$) is poor for astrophysical standards.

- Combination of position, distance and time can lead to association with an electromagnetic detection (e.g. GRB).

**Taking into account spins**

- The inclusion of spin adds significantly to the number of dimensions (9–12–15) and introduces (strong) correlations.

- Failing to take into account spin can result to biases in especially mass parameters.
## Conclusions (numbers are preliminary)

### Using astrophysical knowledge for GW data analysis: no spins

- Knowing the sky position of a source improves determination of:
  - distance ($\sim 20 - 50\%$)
  - inclination
- Knowing the position *and distance* improves inclination further, also in 1-detector analysis

### Using astrophysical knowledge for GW data analysis: spins

- Knowing the sky position of a source improves determination of:
  - distance ($\sim 50\%$)
  - inclination, polarisation angle ($50 - 90\%$)
  - masses ($\sim 20\%$)
  - spin angles
- Knowing the position *and distance* improves:
  - spin magnitude ($\sim 20\%$)
End...
### Predicted detection rates

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Estimates assume $M_{NS} = 1.4 M_\odot$ and $M_{BH} = 10 M_\odot$

CBC group, rates document
MCMC analyses

MCMC parameters

Masses: \( \mathcal{M} \equiv (M_1 + M_2) \eta^{3/5} \) & \( \eta \equiv \frac{M_1 M_2}{(M_1 + M_2)^2} \), distance: \( \log d_L \), time and phase at coalescence: \( t_c \) & \( \varphi_c \), position: \( \alpha \) & \( \sin \delta \), spin magnitude: \( a_{\text{spin}_{1,2}} \), spin orientation: \( \cos \theta_{\text{spin}_{1,2}} \) & \( \varphi_{\text{spin}_{1,2}} \) & binary orientation: \( \cos(\iota) \) & \( \psi \)

MCMC set-up

- \( \geq 5 \) serial chains per run, starting from offset parameter values
- Chain length: \( \sim \) few \( \times 10^6 \) states; burn-in: \( \sim \) few \( \times 10^5 \) states
- Run time: 10 days on a 2.8 GHz CPU for 1.5-pN waveform; \( \sim 2.5 \times \) longer for 3.5-pN

Analysis details: BH-NS signal

Signals injected in simulated noise for H1L1V @ SNR \( \approx 17.0 \)

Fiducial binary: \( M_1, 2 = 10 + 1.4 M_\odot \), \( d_L = 16–23 \) Mpc

Spin: \( a_{\text{spin}} = 0.0, 0.1, 0.5, 0.8, \theta_{\text{SL}} = 20^\circ, 55^\circ \)
MCMC analyses

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- Spin: $a_{\text{spin}} = 0.0, 0.1, 0.5, 0.8$, $\theta_{\text{SL}} = 20^\circ, 55^\circ$
Convergence of chains

- Dots: starting values
- Dashes: injection values
Analysis of a BH-BH signal with spins

van der Sluys et al., in preparation
The nuisance of having spins in your analysis

Signal **without** spins, analysis with spinning template

Signal **with** spins, analysis with spinning template