Constraining the equation of state using advanced gravitational wave detectors

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Overview: EOS effect on binary neutron star waveforms

![Graph showing waveforms for different detector sensitivities: PN INSPIRAL, Initial LIGO: SNR ~ 2, Advanced LIGO: SNR ~ 30, ET: SNR ~ 500, 1.35–1.35 M\text{optimal} \text{ at } 100 \text{ Mpc}]

- **PN INSPIRAL**
- **Initial LIGO: SNR ~ 2**
- **Advanced LIGO: SNR ~ 30**
- **ET: SNR ~ 500**
- **1.35–1.35 \text{Moptimal} \text{ at } 100 \text{ Mpc}**
Overview: EOS effect on binary neutron star waveforms

Leading tidal contribution to inspiral
$\Delta \text{SNR} \approx 1$
up to 450 Hz in AdLIGO

$1.35-1.35$ $M$ optimal @ 100 Mpc
Overview: EOS effect on binary neutron star waveforms

Numerical waveforms
SNR ~ 2
in Broadband AdLIGO

$2f^{1/2} |\tilde{h}(f)|$ (Hz$^{-1/2}$)

$10^{-21}$
$10^{-22}$
$10^{-23}$
$10^{-24}$

$f$ (Hz)

$10^{-21}$
$10^{-22}$
$10^{-23}$
$10^{-24}$

$1.35-1.35 M_{\odot}$ optimal @ 100 Mpc
Preliminary: Equations of state for neutron star cores

Matter in neutron stars compressed to $\sim 1$-10 times nuclear density
($\sim 2$-$20 \times 10^{14}$ g/cm$^3$)

Uncertain pressure of ground state at high density: many-body problem with strong interactions

- Hebeler et. al., Constraints on Neutron Star Radii Based on Chiral Effective Field Theory Interactions, PRL 105, 161102 (2010)
- shaded band shows estimate of current uncertainties
Resulting uncertainty in neutron star structure

different pressure-density relationships give different mass-radius relationship

from Hebeler et. al.
1) Matter effects in a binary system

Consider two extended bodies in orbit or free-fall:

\[ R \]
\[ r \]

Residual gravitational effect is tidal deformation.
Amount of deformation depends on size and matter properties.
Deformations induce changes in the gravitational potential.
Tidal deformability $\lambda$ for realistic EOS

$$\lambda = \frac{Q}{E} = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

$$\lambda = \frac{2}{3}k_2R^5$$

Calculate via linear $Y_{20}$ perturbation of spherical neutron star
$Q$ and $E$ defined by external field of perturbed star
leading terms $\sim r^2$ and $\sim r^{-3}$ when far from star

For given realistic EOS, $\lambda$ is function of $M$
(similar to radius or moment of inertia)
Neutron star structure from simplified EOS variation

\[ \Gamma = 3, \log(p_1) = 34.5 \text{ dyn/cm}^2 \]
Neutron star structure from simplified EOS variation

$$\Gamma = 3, \log(p_1) = 34.4 \text{ dyn/cm}^2$$
Neutron star structure from simplified EOS variation

\[ \Gamma = 3, \log(p_1) = 34.3 \text{ dyn/cm}^2 \]
Neutron star structure from simplified EOS variation

\[ \Gamma = 2.7, \ \log(p_1) = 34.3 \text{ dyn/cm}^2 \]
Γ = 2.7, \log(p_1) = 34.4 \text{ dyn/cm}^2
Neutron star structure from simplified EOS variation

\( \Gamma = 2.7, \log(p_1) = 34.5 \text{ dyn/cm}^2 \)
Neutron star structure from simplified EOS variation

\[ \Gamma = 3.0, \ \log(p_1) = 34.4 \text{ dyn/cm}^2 \]
\( \Gamma = 3.0, \log(p_1) = 34.4 \text{ dyn/cm}^2 \), modified crust
R and $\lambda$ EOS parameter space

![Plot from B. Lackey](image)
Effect of $\lambda$ on waveform

Incorporate first order tidal correction to post-Newtonian waveform

\[ E = \text{Energy of system} = -\frac{1}{2} Mc^2 \eta \times (1 + [\text{PN}]) \]

\[ \dot{E} \text{ (from GW)} = \frac{32}{5} \frac{c^5}{G} \eta^2 \times^5 (1 + [\text{PN}]) \]

\[ x \sim \frac{M}{r} \]
\[ M = m_1 + m_2 \]
\[ \eta = \frac{m_1 m_2}{M^2} \]

Evolve orbit using balance of luminosity and orbital energy
Effect of $\lambda$ on waveform

Incorporate first order tidal correction to post-Newtonian waveform

\[ E = \text{Energy of system} = -\frac{1}{2} M c^2 \eta x \left( 1 + [\text{PN}] - \frac{1}{2} Q_{ij}^1 \mathcal{E}_{ij}^2 + 2 \leftrightarrow 1 \right) \]

\[ \dot{E} \text{ (from GW)} = \frac{32}{5} c^5 G \eta^2 x^5 \left( 1 + [\text{PN}] - \frac{1}{5} \langle Q_{ij}^1 Q_{ij}^1 \rangle + 2 \leftrightarrow 1 \right) \]

$x \sim \frac{M}{r}$

$M = m_1 + m_2$

$\eta = \frac{m_1 m_2}{M^2}$

Evolve orbit using balance of luminosity and orbital energy
Effect of $\lambda$ on waveform

Incorporate first order tidal correction to post-Newtonian waveform

\[ E = \text{Energy of system} = -\frac{1}{2} M c^2 \eta x \left( 1 + [\text{PN}] - 9 \frac{m_2}{m_1} \lambda_1 \frac{x^5}{M^5} + 2 \leftrightarrow 1 \right) \]

\[ \dot{E} \text{ (from GW)} = \frac{32}{5} \frac{c^5}{G} \eta^2 x^5 \left( 1 + [\text{PN}] + 6 \left( \frac{M}{m_1} + 2 \frac{m_2}{m_1} \right) \lambda_1 \frac{x^5}{M^5} + 2 \leftrightarrow 1 \right) \]

\[ x \sim \frac{M}{r} \quad M = m_1 + m_2 \quad \eta = \frac{m_1 m_2}{M^2} \]

Evolve orbit using balance of luminosity and orbital energy
Measuring tidal deformability $\lambda$

*shaded:* Uncertainty in estimating $\lambda$ for Advanced LIGO and ET using “clean” waveform: below 450 Hz only

Hinderer et. al. 2010
0911.3535
Extrapolating into messy region...

Leading order effect: lower bound on EOS contribution
NLO amplification 17% @ 400Hz → 30% @ 1000Hz

NLO in Vines, Hinderer, Flanagan 1101.1673
2) Signal from merger of binary neutron stars

Numerical waveforms
SNR ~ 2
in Broadband AdLIGO

$2f^{1/2} |\tilde{h}(f)| \text{ (Hz}^{-1/2}\text{)}$

1.35–1.35 $M_\odot$ optimal @ 100 Mpc

$f$ (Hz)
Systematic EOS exploration in BNS simulation

vary pressure scale

vary $\Gamma$
Numerical simulation of systematically varied EOS


- initial data: Taniguchi quasiequilibrium code
- Range of EOS
- 1.35-1.35 $M_\odot$ equal mass binary
  - Observed NS masses 1.2-1.5 $M_\odot$ in DNS "average" 1.35 $M_\odot$, mass ratio $> 0.8$
Merger waveform agreement

Aligned using time and phase of peak amplitude

$R = 15.23 \text{ km}, \ \lambda = 11.0 \times 10^{36} \text{ g cm}^2 \text{ s}^2$
Merger waveform agreement

Aligned using time and phase of peak amplitude
Merger waveform agreement

Aligned using time and phase of peak amplitude

$R = 11.61 \text{ km}$, $\lambda = 1.99 \times 10^{36} \text{ g cm}^2 \text{ s}^2$
Merger waveform agreement

Aligned using time and phase of peak amplitude
Merger waveform agreement

Aligned using time and phase of peak amplitude
Range of signals from varying EOS

$\sqrt{S_n(f)}$ and $2\sqrt{\int f |h(f)|^2}$

- EOS 2H
- EOS H
- EOS HB
- EOS B
- EOS B''

f (Hz)

@ 100 Mpc

 BH–BH

AdvLIGO

ET
Measurability estimates: EOS effects on late inspiral

Ideal scenario: Detected strong waveform signal from early PN region

1. Construct hybrid waveforms: connect analytic waveform (with tidal contributions) to numerical inspiral.
2. Early PN sets alignment of waveforms: fixed $t_c$, $\phi_c$
3. Late inspiral/merger differences between EOS integrated against noise curve determine distinguishability.

Measurability estimates require parameterization of signal: e.g. by EOS parameters, radius, $\lambda$, ...
Previously, on arXiv:0901.3258

Point particle post-Newtonian and short numerical inspiral: EOS variation only above 750 Hz

\[ \Delta R \sim 1 \text{ km} \quad \Delta \lambda \sim 1 \times 10^{36} \text{ g cm}^{-2} \text{ s}^{-2} \quad \text{in Broadband Advanced LIGO} \]
\[ \Delta R \sim 0.1 \text{ km} \quad \Delta \lambda \sim 0.1 \times 10^{36} \text{ g cm}^{-2} \text{ s}^{-2} \quad \text{in ET} \]

For \( R \) around 12 km and \( \lambda \) around 3
Measurability estimates: EOS effects on end of inspiral

New, long waveforms allow numerical-only estimate:
Slide noise-weighted numerical waveforms against each other to minimize differences.
Measurability estimates: EOS effects on end of inspiral

\[
\langle \Delta p \rangle_{p_{\text{avg}}} \approx \frac{p_1 - p_2}{\langle h(p_1) - h(p_2) | h(p_1) - h(p_2) \rangle^{1/2}}
\]

Expected result for coarsely sampled parameter:
100Hz-aligned post-Newtonian + $\lambda$ of simulated EOS

(no frequency cutoff)
Measurability estimates: EOS effects on end of inspiral

\[
\langle \Delta p \rangle \mid_{p_{\text{avg}}} \simeq \frac{p_1 - p_2}{\langle h(p_1) - h(p_2) \rangle^{1/2}}
\]

Using best-aligned numerical waveforms only, parameterized by \( \lambda \) of simulated EOS

![SNR of waveform difference vs. difference in \( \lambda \) (10^{36} \text{g cm}^2 \text{ s}^2)]
Measurability estimates: EOS effects on end of inspiral

\[
\langle \Delta p \rangle \bigg|_{p_{\text{avg}}} \approx \frac{p_1 - p_2}{\langle h(p_1) - h(p_2) \mid h(p_1) - h(p_2) \rangle^{1/2}}
\]

\( \delta R, R \) is radius of isolated neutron star

<table>
<thead>
<tr>
<th>( R )</th>
<th>Broadband AdLIGO</th>
<th>ET-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.8</td>
<td>±0.9 km</td>
<td>±0.09 km</td>
</tr>
<tr>
<td>10.9</td>
<td>±1.2 km</td>
<td>±0.15 km</td>
</tr>
<tr>
<td>11.3</td>
<td>±1.2 km</td>
<td>±0.16 km</td>
</tr>
<tr>
<td>11.6</td>
<td>±1.2 km</td>
<td>±0.16 km</td>
</tr>
<tr>
<td>11.9</td>
<td>±0.8 km</td>
<td>±0.10 km</td>
</tr>
</tbody>
</table>

Systematics within same EOS from numerical simulation \( \sim 0.1 \) km
Other sources: imperfect parameterization, approximation of waveform derivative
Independent measurability of merger and post-merger?

Waveforms after merger:

Equal mass @ 100 Mpc
Post–merger only

Hypermassive Remnant  ET SNR  Prompt collapse  ET SNR
EOS 2H  \(\sim 6\)  EOS B  \(\sim \frac{1}{2}\)
EOS H  \(\sim 3\)  EOS B”  \(\sim 1\)
EOS HB  \(\sim 3\)

These estimates are very dependent on the uncertain stability of the PMO.
## Linking regions

Two measurability estimates

<table>
<thead>
<tr>
<th>Modified PN waveform</th>
<th>Numerical simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>information below 450 Hz</td>
<td>information above 500 Hz</td>
</tr>
<tr>
<td>before other effects at higher frequency</td>
<td>after start of numerical simulation</td>
</tr>
</tbody>
</table>

Accurate hybrid waveforms incorporating information from both regions would have comparable sensitivity to extrapolated PN: \( \lambda \sim 1 \) or less measurable
EOS parameters from black hole/neutron star waveforms

Inspiral effects - see talk of F. Pannarale after coffee break
Merger estimates - from B. Lackey, K. Kyutoku, M. Shibata, P. Brady, J. Friedman

- Cutoff frequency from tidal disruption and
- Phase shift during ringdown

\[(q = 2, M_{\text{NS}} = 1.35M_\odot) \text{ at } 100 \text{ Mpc}\]
EOS parameters from black hole/neutron star waveforms

Inspiral effects - see talk of F. Pannarale after coffee break
Merger estimates - from B. Lackey, K. Kyutoku, M. Shibata, P. Brady, J. Friedman

Errors shown for $D_{\text{eff}} = 100$ Mpc and Einstein Telescope-B noise curve
Errors about 5-10 times larger for Advanced LIGO