Multi-Detector Line Veto for Continuous-Wave Searches

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Abstract

Coherently combining data from different detectors increases the sensitivity of searches for continuous gravitational waves (CW) in pure Gaussian stationary noise. It does not, however, discriminate against commonly present single-detector artifacts that resemble CW signals ("lines"). We present an extended multi-detector statistic that incorporates a coincidence veto against such line artifacts, using Bayesian model selection, and we re-derive a Frequentist ad-hoc coincidence veto as a special case. We show Monte-Carlo results testing the efficiency of this method in simulated Gaussian noise with injected line-artifacts.

Preliminaries and Notation

- We present a "first-stage" Bayesian veto method against quasi-stationary line artifacts ("lines") that can trigger CW templates.
- Similar ideas have been explored previously in the context of CBC [4] and burst searches [5, 6].
- We denote boldface $\boldsymbol{x}(t)$ the vector of observed data $x_X(t)$ from different detectors X, e.g. for two detectors $\boldsymbol{x}(t) = \{x_1(t), x_2(t)\}$.
- The multi-detector signal template with unknown amplitude parameters $\mathcal{A}$ is denoted as $\boldsymbol{h}(t;\mathcal{A})$. For simplicity we assume all other signal parameters (frequency, sky-position, . . .) as *known*.

Standard CW coherent multi-IFO detection statistic

The standard coherent multi-detector CW detection statistic compares the two hypotheses:

$\mathcal{H}_G$: pure Gaussian noise: $\qquad \boldsymbol{x}(t) = \boldsymbol{n}(t)$

$\mathcal{H}_S$: Gaussian noise *plus* a *signal* $\mathcal{A}$: $\qquad \boldsymbol{x}(t) = \boldsymbol{n}(t) + \boldsymbol{h}(t;\mathcal{A})$

☞ Standard odds ratio:

$$O_{SG}(\boldsymbol{x}) \equiv \frac{P(\mathcal{H}_S|\boldsymbol{x})}{P(\mathcal{H}_G|\boldsymbol{x})} \propto \int \mathcal{L}(\boldsymbol{x};\mathcal{A})\, P(\mathcal{A}|\mathcal{H}_S)\, d\mathcal{A} \qquad (1)$$

where $\mathcal{L}(\boldsymbol{x};\mathcal{A})$ is the usual likelihood ratio:

$$\mathcal{L}(\boldsymbol{x};\mathcal{A}) \equiv \frac{P(\boldsymbol{x}|\mathcal{H}_S\mathcal{A})}{P(\boldsymbol{x}|\mathcal{H}_G)}$$

Using uniform "$\mathcal{F}$-statistic priors" [3] on $\mathcal{A}$, the marginalization integral (1) is analytic, and we obtain

$$O_{SG}(\boldsymbol{x}) \propto e^{\mathcal{F}(\boldsymbol{x})} \qquad (2)$$

in terms of the standard multi-detector $\mathcal{F}$-statistic $\mathcal{F}(\boldsymbol{x})$, which was first derived by a Frequentist maximum-likelihood method [1, 2].

Extended noise hypothesis to veto detector artifacts ("lines")

Problem with $O_{SG}(\boldsymbol{x}) \propto e^{\mathcal{F}(\boldsymbol{x})}$:

Anything that looks more like $\mathcal{H}_S$ rather than $\mathcal{H}_G$ will result in large values for O_{SG}, regardless of its "goodness-of-fit" to $\mathcal{H}_S$!
In particular: quasi-monochromatic, stationary detector artefacts ("lines") that resemble the CW signal family $\boldsymbol{h}(t;\mathcal{A})$ can cause large outliers.

Solution: add an alternative hypothesis $\mathcal{H}_L$ that fits lines *better* than $\mathcal{H}_S$
Modeling general lines will be difficult, as the line morphology of detectors is not well known or understood.
However: we only care about lines that trigger $\mathcal{H}_S$, so we can simply use the signal model to build a coincidence veto against lines!

☞ define a "Zeroth order line" $\equiv$ non-coincident signal trigger

$\mathcal{H}_L = \{\boldsymbol{x}$ looks like a signal in only one detector$\}$

$\mathcal{H}_L^X : x_X(t) = n_X(t) + h_X(t;\mathcal{A}) \qquad$ for detector X

For two detectors $X = 1, 2$:

$$\mathcal{H}_L : \left(\mathcal{H}_L^1 \text{ and } \mathcal{H}_G^2\right) \text{ or } \left(\mathcal{H}_G^1 \text{ and } \mathcal{H}_L^2\right)$$

$$\Longrightarrow\; P(\mathcal{H}_L) = P\left(\mathcal{H}_L^1\right) P\left(\mathcal{H}_G^2\right) + P\left(\mathcal{H}_G^1\right) P\left(\mathcal{H}_L^2\right) \qquad (3)$$

Line-vetoing statistics

Using uniform "$\mathcal{F}$-statistic priors" on $\mathcal{A}$, one finds

$$P(\mathcal{H}_L|\boldsymbol{x}) = P(\mathcal{H}_G|\boldsymbol{x})\; c_0 \left[l_1\, e^{\mathcal{F}_1(x_1)} + l_2\, e^{\mathcal{F}_2(x_2)}\right]$$

with *line density* $l_X \equiv P\left(\mathcal{H}_L^X\right)/P\left(\mathcal{H}_G^X\right)$ in detector X.

There are (at least) *two* useful ways to use the line hypothesis $\mathcal{H}_L$:

- as follow-up line-veto for "loud" $\mathcal{F}$-stat candidates ($\mathcal{H}_G$ ruled out):

$$O_{SL}(\boldsymbol{x}) \equiv \frac{P(\mathcal{H}_S|\boldsymbol{x})}{P(\mathcal{H}_L|\boldsymbol{x})} \propto \frac{e^{\mathcal{F}(\boldsymbol{x})}}{l_1\, e^{\mathcal{F}_1(x_1)} + l_2\, e^{\mathcal{F}_2(x_2)}} \qquad (4)$$

- as a stand-alone "line-robust" detection statistic

$$O_{SN}(\boldsymbol{x}) \equiv \frac{P(\mathcal{H}_S|\boldsymbol{x})}{P(\mathcal{H}_L|\boldsymbol{x}) + P(\mathcal{H}_G|\boldsymbol{x})} = \left[O_{SN}^{-1}(\boldsymbol{x}) + O_{SG}^{-1}(\boldsymbol{x})\right]^{-1} \qquad (5)$$

Recovering the *ad-hoc* $\mathcal{F}$-consistency veto

In the special case $l_1 = l_2$, the line veto O_{SL} of (4) can be expressed as:
$\ln O_{SL}(\boldsymbol{x}) - \ln O_{SL}{}^{(0)} \approx \mathcal{F}(\boldsymbol{x}) - \max\{\mathcal{F}_1(x_1), \mathcal{F}_2(x_2)\}$

We therefore recover the *ad-hoc* veto criterion as a special case, namely

$$\text{veto if } \max\{\mathcal{F}_1, \mathcal{F}_2\} > \mathcal{F}(\boldsymbol{x}) \iff \left[\ln O_{SL}(\boldsymbol{x}) - \ln O_{SL}{}^{(0)}\right] < 0 \qquad (6)$$

Preliminary Monte-Carlo tests on veto statistics

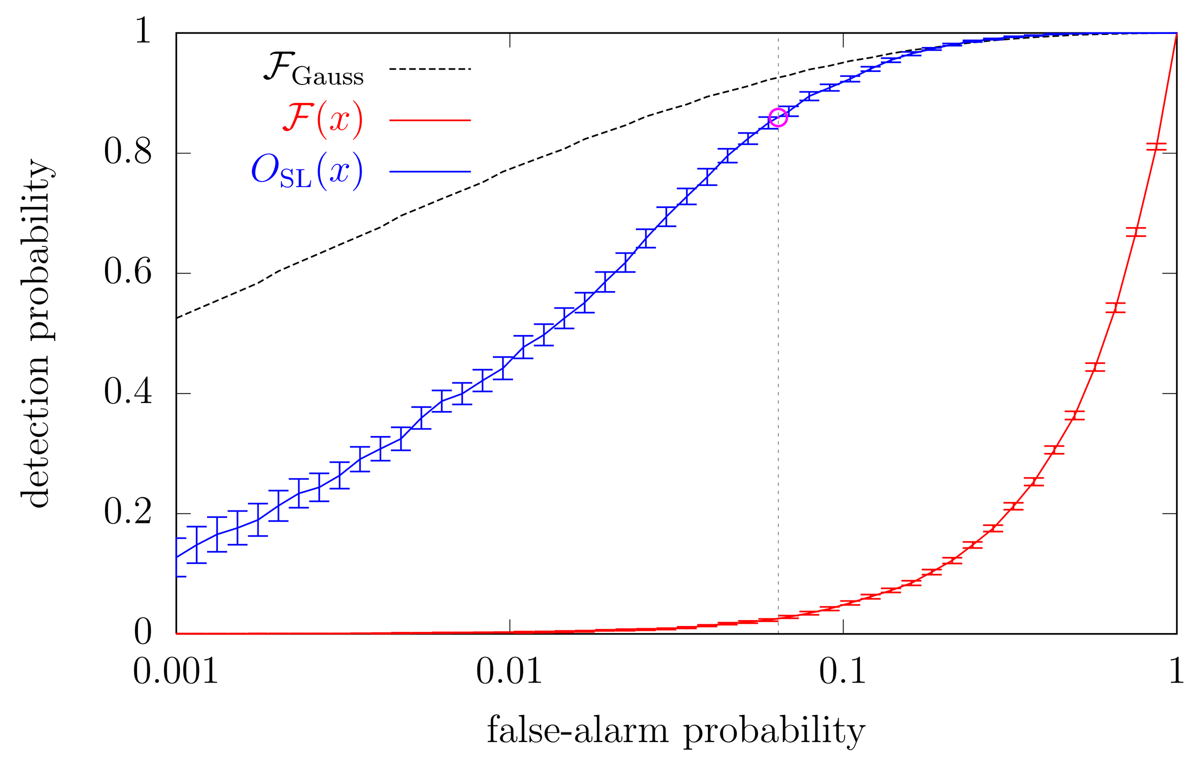

Figure: Line-vetoing power of $O_{SL}(\boldsymbol{x})$ of (4) and multi-IFO $\mathcal{F}(\boldsymbol{x})$ of (2), for candidates that are *either* signals $\mathcal{H}_S$ *or* lines $\mathcal{H}_L$ (both with fixed SNR=4). The circle indicates the performance of the ad-hoc veto (6). $\mathcal{F}_{\rm Gauss}$ is the theoretical $\mathcal{F}(\boldsymbol{x})$ performance in $\mathcal{H}_G$.

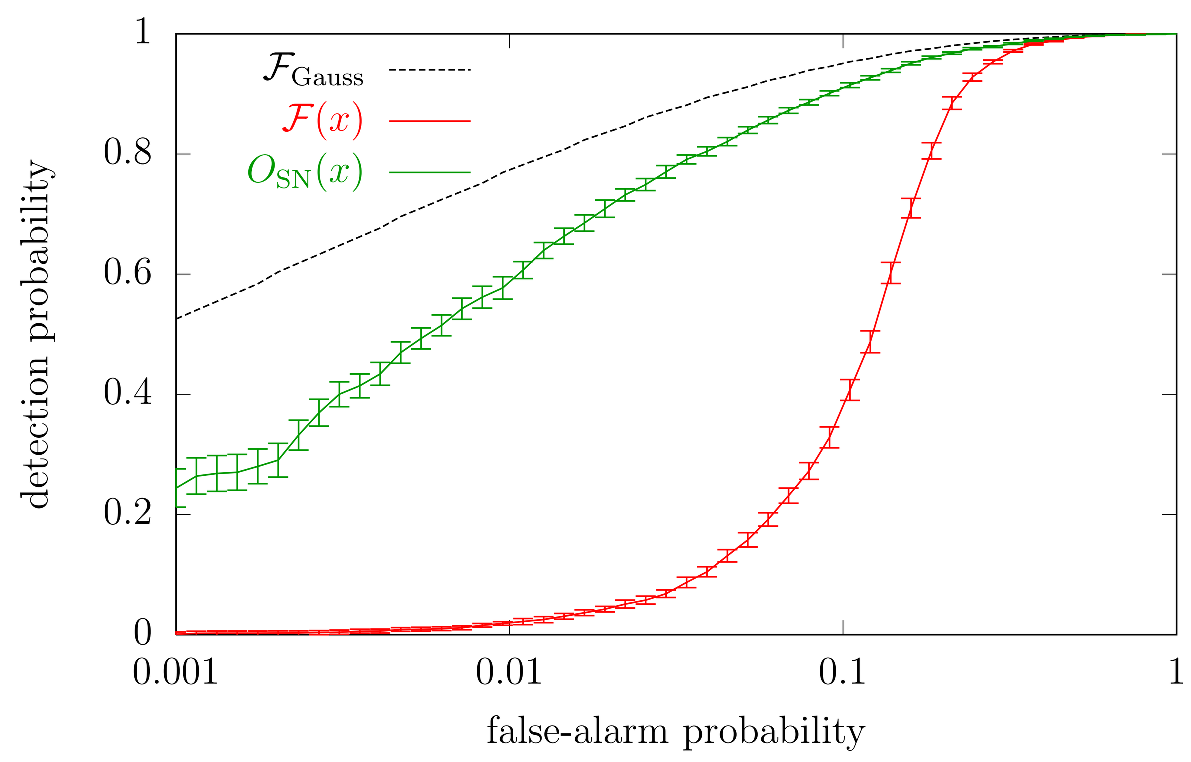

Figure: Detection power of $O_{SN}(\boldsymbol{x})$ of (5) and $\mathcal{F}(\boldsymbol{x})$ of (2), for data containing either a signal $\mathcal{H}_S$ (SNR=4), pure Gaussian noise $\mathcal{H}_G$ or with 20% probability a line $\mathcal{H}_L$ (SNR=4).

Future plans

- more extensive Monte-Carlo studies on simulated data
- test veto on real detector noise and on Einstein@Home candidates
- allow for coincident lines (different $\mathcal{A}_1$ and $\mathcal{A}_2$)
- extend to "first order" line-models: stationary sinusoids

References

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