Cross-Correlation Searches for Periodic Gravitational Waves

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Abstract

Cross-correlation of gravitational-wave (GW) data streams have been used to search for stochastic backgrounds, and the same technique was applied to look for periodic GWs from the low-mass X-ray binary (LMXB) Sco X-1. A technique has been developed which reﬁnes the cross-correlation scheme to take full advantage of the signal model for periodic gravitational waves from rotating neutron stars. By varying the time window over which data streams are correlated, the search can “trade off” between parameter sensitivity and computational cost. Possible search targets include SN1987A remnant and Sco X-1.

Cross-Correlation for Stochastic Signals

Cross-correlation is a standard technique to search for faint signal in noise:

\[
x_1(t) = n_{1}(t) + h(t) = n_{1}(t) + \tilde{h}(t) \cdot \tilde{d}_1(t)
\]

\[
x_2(t) = n_{2}(t) + h(t) = n_{2}(t) + \tilde{h}(t) \cdot \tilde{d}_1(t)
\]

Application to stochastic background:[1]: expectation value due to correlations in random signals

\[
\langle \tilde{g}\tilde{s}_1(f) \tilde{s}_2(f) \rangle = \langle \tilde{h}(f) \tilde{h}(f) \rangle = \delta(f-f')\gamma_{\mathrm{sft}}(f/2)
\]

\[
\gamma_{\mathrm{sft}}(f) = \text{GW spectrum}
\]

\[
\tilde{g}_{ij}(\tau) = \left\langle \tau \right\rangle_{\text{obs}} \propto \cos(\psi(\tau))\gamma_{\mathrm{sft}}(f/2)
\]

\[
\tilde{g}_{ij}(\tau) = \langle \tilde{h}(f) \rangle \tilde{s}_{\text{obs}}(f)
\]

Used to search for pointlike stochastic sources[2] including Scorpius X-1,[3].

Cross-Correlation for Periodic Signals

Sco X-1 not random emitter; low-mass X-ray binary: neutron star in binary orbit w/companion. GW signal from rotating neutron star:

\[
\tilde{h}(t) = \tilde{h}_0 \left[ 1 + \cos^2 \frac{\pi}{2} \cos(\Phi(t)) \right] \tau_\text{c} + \cos(\psi(\tau)) \gamma_{\mathrm{sft}}(f/2)
\]

\[
\cdot \psi: \text{inclination of NS spin}
\]

\[
\cdot \Phi(t): \text{phase evolution in rest frame;}
\]

\[
\cdot \psi(\tau): \text{Doppler mod from detector motion (6 binary orbit)}
\]

Include features of signal in cross-cor method:

\[
\cdot \text{Long-term coherence: can cross-correlate data from different times}
\]

\[
\cdot \text{Doppler shift @ detector: correlations peaked @ different freqs}
\]

Note signal cross-correlation deterministic

\[
\langle \tilde{x}_1(f) \tilde{x}_2(f) \rangle = \tilde{h}_0 \tilde{h}_0 \delta_\text{obs}(f) - f_0 - f_0 - f_0 + f_0)
\]

\[
\cdot h_0(t) = \text{Short Fourier Transform, duration T}_\text{obs}
\]

Theoretical Sensitivity

Amplitude sensitivity of combined statistic:

\[
h_0^\text{obs} \propto \left[ \sum_{i,j} \left| \tilde{g}_{ij} \right|^2 \right]^{-1/2} T_{\text{obs}}^{1/2}
\]

\[
\text{If all pairs included, } N_{\text{pairs}} \propto N_{\text{obs}}
\]

\[
h_0^\text{obs} \propto \left( N_{\text{obs}} T_{\text{obs}} \right)^{1/2} = T_{\text{obs}}^{1/2}
\]

Coherent search

\[
\cdot \text{If only simultaneous pairs, } N_{\text{pairs}} \propto N_{\text{obs}}
\]

\[
h_0^\text{obs} \propto N_{\text{obs}}^{1/2} T_{\text{obs}}^{1/2} = T_{\text{obs}}^{1/4}
\]

\[
\cdot \text{If only pairs separated by } T_{\text{sep}} \text{ or less, } h_0^\text{obs} \propto T_{\text{obs}}^{1/4} T_{\text{sep}}^{1/4}
\]

Can simplify sensitivity estimates if observations uniformly distributed in sidereal time:

\[
\langle \tilde{g}_{ij}(f) \rangle_\text{obs} \propto \cos(\psi(\tau))
\]

Application: Supernova 1987A Remnant

SN1987A remnant likely contains young neutron star, rapidly spinning down[5, 6]. Can search for GW w/cross-correlation method. Need to search over frequency and spinwands; rather than searching J1, f1, J2, f2, . . . , use phase model w/GW spinwands x fâ¬‘ and EM spinwands x fâ¬‘.

Search over J1, f1, QGW, QEM, n1.

Can ballpark sensitivity using initial LIGO design & assuming only simultaneous LLO and LHO data are used. Compares favorably to indirect age-based limit h0 < 3.4 x 10^-2;

References