Motivation

- Gravitational wave (GW) detectors like Advanced LIGO need waveform templates for optimal sensitivity.
- Usual approach: Hybrid waveforms, composed of a post-Newtonian (PN) inspiral and a numerical relativity (NR) late inspiral, merger and ringdown.
- Both NINJA and NRAR collaborations require knowledge of hybrid-errors to ensure sufficient accuracy.
- Our goal: Comprehensive error analysis of hybrid waveforms.
  - Choice of matching region of the PN and NR waveforms.
  - Systematic errors in NR waveform.
  - Choice of PN approximants.
- Current NR waveforms are likely too short.

Calculating waveform differences

- Consider a waveform template \( h \) which differs from the exact waveform by an amount \( \delta h \neq 0 \). The error \( \delta h \) does not affect data-analysis if [4]:

\[
\langle \delta h, \delta h \rangle < \frac{0.01 \text{(SNR)}^2}{1, \text{ param. estimation}}
\]

- Define error criterion \( ||\delta h||/||h||\):

\[
\frac{||\delta h||}{||h||} = \frac{0.1}{1/\text{(SNR), param. estimation}}
\]

- Properties of \( ||\delta h||/||h||\):
  - linear in error \( \delta h \)
  - independent of source distance
  - unifies bounds for event detection and param. estimation

- Approximate \( \delta h \) by difference between low-quality hybrid and higher-quality “reference hybrid”, to estimate error of low-quality hybrid.

\( ||\delta h||/||h|| \) depends on total mass \( M \), therefore output is plot of \( ||\delta h||/||h|| \) vs. \( M \).

Results I: Systematic Errors in the NR Waveform

- Systematic errors in the NR waveform will affect the hybrid.

- Illustration: waveform extracted at finite radius. This leads to systematic phase-errors at low frequencies (right figure).

- Sufficient for GW detection, insufficient for parameter estimation at \( \text{SNR} > 18 \).
- Even when NR waveform seems sufficient for parameter estimation, hybrid isn’t necessarily.

Results II: Effect of Matching Region and PN approximant

- Compare hybrid matched at high frequency \( M_{\text{u}} \) with reference hybrid matched at a lower frequency.
- Hybridization at lower \( M_{\text{u}} \) decreases error in hybrid (as expected).
- Error also depends on difference between trial \( M_{\text{u}} \) and reference \( M_{\text{u}} \).

- As reference \( M_{\text{u}} \) decreases, we asymptote towards error between true gravitational waveform and hybrid.

- Assume true waveform is hybrid of NR and Taylor T4 PN waveforms.

- Hybrids constructed with Taylor T3 waveforms are insufficient for detection.

- Hybrids constructed with Taylor T4 waveforms are sufficient for parameter estimation at a matching interval of [0.064 ± 0.0032].

- Note: TaylorT4 is known to agree unreasonably well for this special case (equal mass, no spin). The behavior of TaylorT3 is more representative for generic parameters.

Conclusions

- Quantifying errors of hybrid waveforms is difficult.
  - Depending on total mass, different portions of the hybrid are in the GW-detector frequency band.
  - There are many potentially relevant sources of error.

- NR with 15 orbits likely not long enough for hybrids:
  - Need a longer NR waveform if matching with Taylor T3.
  - Taylor T4 hybrids work well for simple case, but not necessarily in general.

- Future work:
  - Extend to non-zero spins and unequal mass ratios.

References:


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