Summary

Statistical techniques can be used to gain insight into binary black hole inspiral dynamics in the post-Newtonian (PN) approximation. We sample initial spin orientations of equal-mass, equal-spin binaries and study predictive properties of other new spin variables.

Large Parameter Space

The binary black hole inspiral parameter is high-dimensional. Even for circular orbits, a full characterization of the binary requires specification of the two masses $m_1$ and $m_2$, and the two spin-vectors $S_1$ and $S_2$, giving 8 dimensions. We exploit the total mass scale-free nature of GR and work with a 7-dimensional parameter space.

Statistical Sampling

We perform a statistical analysis of the full 7-dimensional parameter space. We focus on identifying the most and least relevant variables in the spin dynamics. In order to identify these variables we use a Principal Component Analysis (PCA) — a standard technique from multi-variate statistics.

Post-Newtonian Approximation

We use the PN equations from Ref. [1] which describe a quasi-circular inspiral up to 3.5PN order in angular frequency $\omega$ and spin effects up to 2PN order with the covariant spin supplementary condition.

Principal Component Analysis

In PCA one seeks to determine the variables that are statistically relevant and to dimensionally reduce the problem by eliminating those that are not. The covariance matrix $C_{ij}$ is given by $\langle (X_i - \langle X_i \rangle)(X_j - \langle X_j \rangle) \rangle$, where $X_i$ ($i = 1, \ldots, n$) are stochastic variables. The brackets represent expectation values computed over the unit sphere.

The covariance matrix $C_{ij}$ is diagonalized and the eigenvectors (or principal components — PCs) are ranked by the size of their eigenvalues $\lambda_i$. The PCs are uncorrelated and $\lambda_i$ is the variance of the PC$^i$. For small $\lambda_i$ the corresponding PC$^i$ is likely to remain close to its mean.

Variables for PCA

We choose combinations of spin and orbital angular momentum scalar products and take the difference between the final $f$ and the initial $i$ state. For example $\Delta L := \Delta(S_1 \cdot L) = S_1 \cdot L_f - S_1 \cdot L_i$.

Spin-Orbit Case

As an illustrative example we perform a PCA for only 2 variables: $\Delta L$ and $\Delta S_1 L$.

This plot shows the variance of the principal component $\Delta L^0_{50}$ as a function of the black hole spin magnitudes $\chi_1, \chi_2$ and their masses (note that $m_1 + m_2 = 1$). Notice that the variance remains small for all parameter values and hence that $\Delta L^0_{50}$ is well conserved throughout the parameter space.

Instantaneous Approach

We have derived an instantaneous approximation to the PCA variables from the PN equations keeping terms up to $O(\Delta t)$. As an example we found:

$$\Delta L = 2m_2 \omega^2 \Delta t \left( L \cdot (S_1 \times \dot{S}_1) \right) \left(1 - \omega^2 \hat{S}_1 \cdot \hat{L}\right).$$

This plot shows the PCs and a scatter plot of the $\Delta L$ and $\Delta S_1 L$ data from 1,000 simulations (the PCs were computed using 100,000 simulations). Note how much of the data is explained by PC$^1$ and how little by PC$^2$.

Conservation of $E_{50}^{NL}$

Further Information

Please see our paper [2], also available from the arXiv at http://arxiv.org/abs/1005.5560

Acknowledgments

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References


Conclusions

• We have performed a statistical study of the binary black hole inspiral problem in the post-Newtonian approximation.

• Spin orientations were randomly sampled with spin magnitudes and individual masses as parameters.

• We found new conserved quantity $\Delta E^NL$, that performs much better than previously known PN based expressions.

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